MATHEMATICS

Chapter 1: Number Systems

Number Systems

1. Numbers 1, 2, 3…….∞, which are used for counting are called **natural numbers**. The collection of natural numbers is denoted by **N.** Therefore, N = {1, 2, 3, 4, 5, ……}.

2. When 0 is included with the natural numbers, then the new collection of numbers called is called **whole number**. The collection of whole numbers is denoted by **W**. Therefore, W = {0, 1, 2, 3, 4, 5, ……}.

3. The negative of natural numbers, 0 and the natural number together constitutes **integers**. The collection of integers is denoted by **I**. Therefore, I = {…, -3, -2, -1, 0, 1, 2, 3, ……}.

4. The numbers which can be represented in the form of p/q, where q ≠ 0 and p and q are integers are called **rational numbers**. Rational numbers are denoted by **Q.** If p and q are co-prime, then the rational number is in its simplest form.

5. All-natural numbers, whole numbers and integer are rational number.

6. **Equivalent rational numbers** (or fractions) have same (equal) values when written in the simplest form.

7. Rational number between two numbers x and y $= \frac{x+y}{2}$.

8. There are infinitely many rational numbers between any two given rational numbers.

9. The numbers which are not of the form of p/ q, where q ≠ 0 and p and q are integers are called irrational numbers. For example: $\sqrt{2}, \sqrt{7}, \pi$, etc.

10. Rational and irrational numbers together constitute are called **real numbers**. The collection of real numbers is denoted by **R.**

11. Irrational number between two numbers x and y

$$= \begin{cases} \sqrt{\sqrt{xy}}, \text{if x and y both are irrational numbers} \\ \sqrt{\sqrt{xy}}, \text{if x is rational number and y is irrational number} \\ \sqrt{xy}, \text{if x } \times \text{ y is not a perfect square and x, y both are rational numbers} \end{cases}$$

12. **Terminating fractions** are the fractions which leaves remainder 0 on division.

13. **Recurring fractions** are the fractions which never leave a remainder 0 on division.

14. The decimal expansion of **rational** number is **either terminating or non-terminating recurring**. Also, a number whose decimal expansion is terminating or non-terminating recurring is rational.

15. The decimal expansion of an **irrational** number is **non-terminating non-recurring**. Also,

a number whose decimal expansion is non-terminating non-recurring is irrational.

16. Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

17. The process of visualization of numbers on the number line through a magnifying glass is known as the process of **successive magnification.** This technique is used to represent a real number with non-terminating recurring decimal expansion.

18. Irrational numbers like $\sqrt{2}, \sqrt{3}, \sqrt{5} \dots \sqrt{n}$, for any positive integer n can be represented on number line by using Pythagoras theorem.

19. If a > 0 is a real number, then $\sqrt{a} = b$ means $b^2 = a$ and b > 0.

20. For any positive real number x, we have:

$$x = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x+1}{2}\right)^2$$

21. For every positive real number x, $\sqrt{x}$ can be represented by a point on the number line using the following steps:

    i. Obtain the positive real number, say x.

    ii. Draw a line and mark a point A on it.

    iii. Mark a point B on the line such that AB = x units.

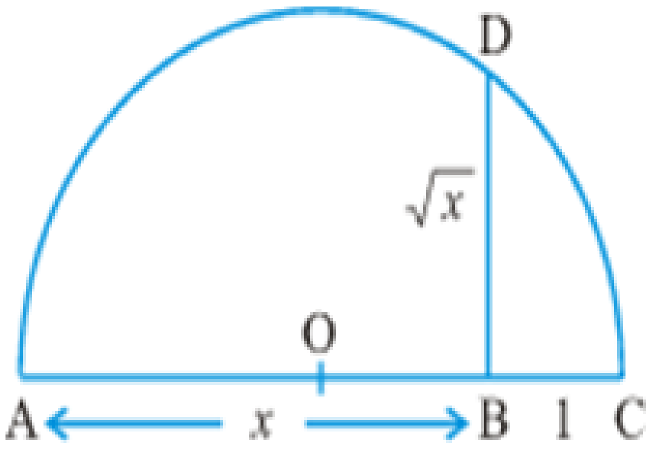
    iv. From B, mark a distance of 1 unit on extended AB and name the new point as C.

    v. Find the mid-point of AC and name that point as O.

    vi. Draw a semi-circle with centre O and radius OC.

    vii. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D.

    viii. Length BD is equal to $\sqrt{x}$.



22. Properties of irrational numbers:

    i. The sum, difference, product and quotient of two irrational numbers need not always be an irrational number.
    ii. Negative of an irrational number is an irrational number.
    iii. Sum of a rational and an irrational number is irrational.
    iv. Product and quotient of a non-zero rational and irrational number is always irrational.

23. Let a > 0 be a real number and n be a positive integer. Then $\sqrt[n]{a} = b$, if b^n = a and b > 0.

    The symbol '$\sqrt{\ }$' is called the **radical sign**.

24. For real numbers a > 0 and b > 0:

    i. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
    ii. $\sqrt{\frac{a}{b}} = \sqrt{\frac{a}{b}}$
    iii. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
    iv. $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{bc} + \sqrt{ad} - \sqrt{bd}$
    v. $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$
    vi. $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

25. The process of removing the radical sign from the denominator of an expression to convert it to an equivalent expression whose denominator is a rational number is called **rationalising the denominator**.

26. The multiplicating factor used for rationalising the denominator is called the **rationalising factor**.

27. If a and b are positive real numbers, then

Rationalising factor of $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$

Rationalising factor of $\frac{1}{a\pm\sqrt{b}}$ is $a \neq \sqrt{b}$

Rationalising factor of $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a} \neq \sqrt{b}$

28. The **exponent** is the number of times the base is multiplied by itself.

29. In the exponential representation a^m, a is called the **base** and m is called the **exponent or power**.

30. **Laws of exponents**: If a, b are positive real numbers and m, n are rational numbers, then

i. $a^m \times a^n = a^{m+n}$
ii. $a^m \div a^n = a^{m-n}$
iii. $\left(a^m\right)^n = a^{mn}$
iv. $a^{-n} = \frac{1}{a^n}$
v. $(ab)^n = a^n b^n$
vi. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
vii. $a^{m/n} = \left(a^m\right)^{1/n} = \left(a^{1/n}\right)^m$ or $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
viii. $a^0 = 1$

Numbers

Number: Arithmetical value representing a particular quantity. The various types of numbers are Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, Real Numbers etc.

Natural Numbers

Natural numbers(N) are positive numbers i.e. 1, 2, 3 ..and so on.

Whole Numbers

Whole numbers (W) are 0, 1, 2, .. and so on. Whole numbers are all Natural Numbers including '0'. Whole numbers do not include any fractions, negative numbers or decimals.

Integers

Integers are the numbers that includes whole numbers along with the negative numbers.

Rational Numbers

A number 'r' is called a rational number if it can be written in the form p/q, where p and q are integers and $q \neq 0$.

Irrational Numbers

Any number that cannot be expressed in the form of p/q, where p and q are integers and $q \neq 0$, is an irrational number. Examples: $\sqrt{2}$, 1.010024563..., e, π

Real Numbers

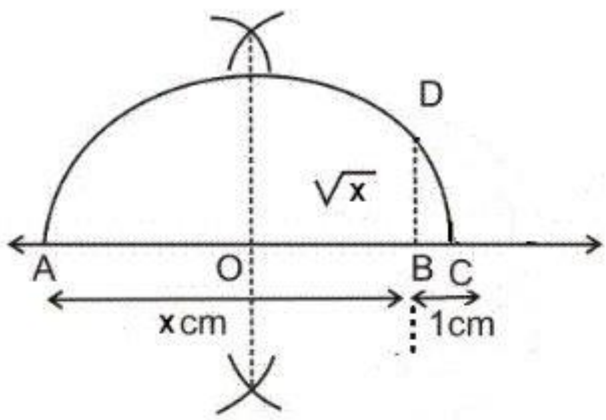
Any number which can be represented on the number line is a Real Number(R). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

Irrational Numbers

Representation of Irrational numbers on the Number line

Let $\sqrt{x}$ be an irrational number. To represent it on the number line we will follow the following steps:

- Take any point A. Draw a line AB = x units.
- Extend AB to point C such that BC = 1 unit.
- Find out the mid-point of AC and name it 'O'. With 'O' as the centre draw a semi-circle with radius OC.
- Draw a straight line from B which is perpendicular to AC, such that it intersects the semi-circle at point D.
- Length of BD = $\sqrt{x}$.



Constructions to Find the root of x.

With BD as the radius and origin as the centre, cut the positive side of the number line to get $\sqrt{x}$.

Identities for Irrational Numbers

Arithmetic operations between:

- rational and irrational will give an irrational number.
- irrational and irrational will give a rational or irrational number.

Example: $2 \times \sqrt{3} = 2\sqrt{3}$ i.e. irrational. $\sqrt{3} \times \sqrt{3} = 3$ which is rational.

Identities for irrational numbers

Rationalisation

Rationalisation is converting an irrational number into a rational number. Suppose if we have to rationalise $1/\sqrt{a}$.

$1/\sqrt{a} \times 1/\sqrt{a} = 1/a$

Rationalisation of $1/\sqrt{a} + b$:

$(1/\sqrt{a} + b) \times (1/\sqrt{a} - b) = (1/a - b^2)$

Laws of Exponents for Real Numbers

If a, b, m and n are real numbers then:

$a^m \times a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$a^m/a^n = a^{m-n}$

$a^m b^m = (ab)^m$

Here, a and b are the bases and m and n are exponents.

Exponential representation of irrational numbers

If a > 0 and n is a positive integer, then: n√a = a1n Let a > 0 be a real number and p and q be rational numbers, then:

$a^p \times a^q = a^{p+q}$

$(a^p)^q = a^{pq}$

$a^p / a^q = a^{p-q}$

$a^p b^p = (ab)^p$

Decimal Representation of Rational Numbers

Decimal expansion of Rational and Irrational Numbers

The decimal expansion of a rational number is either terminating or non- terminating and recurring.

Example: 1/2 = 0.5 , 1/3 = 3.33…….

The decimal expansion of an irrational number is non terminating and non-recurring.

Examples: √2 = 1.41421356..

Expressing Decimals as rational numbers

Case 1 – Terminating Decimals

Example – 0.625

Let x = 0.625

If the number of digits after the decimal point is y, then multiply and divide the number by 10y.

So, x = 0.625 × 1000/1000 = 625/1000 Then, reduce the obtained fraction to its simplest form.

Hence, x = 5/8

Case 2: Recurring Decimals

If the number is non-terminating and recurring, then we will follow the following steps to convert it into a rational number:

Example = $1.0\overline{42}$

Step 1. Let x = $1.0\overline{42}$ ……..(1)

Step 2. Multiply the first equation with 10y, where y is the number of digits that are recurring.

Thus, 100x = $104.\overline{242}$ ………(2)

Steps 3. Subtract equation 1 from equation 2. On subtracting equation 1 from 2, we get99x = 103.2x = 103.2/99 = 1032/990

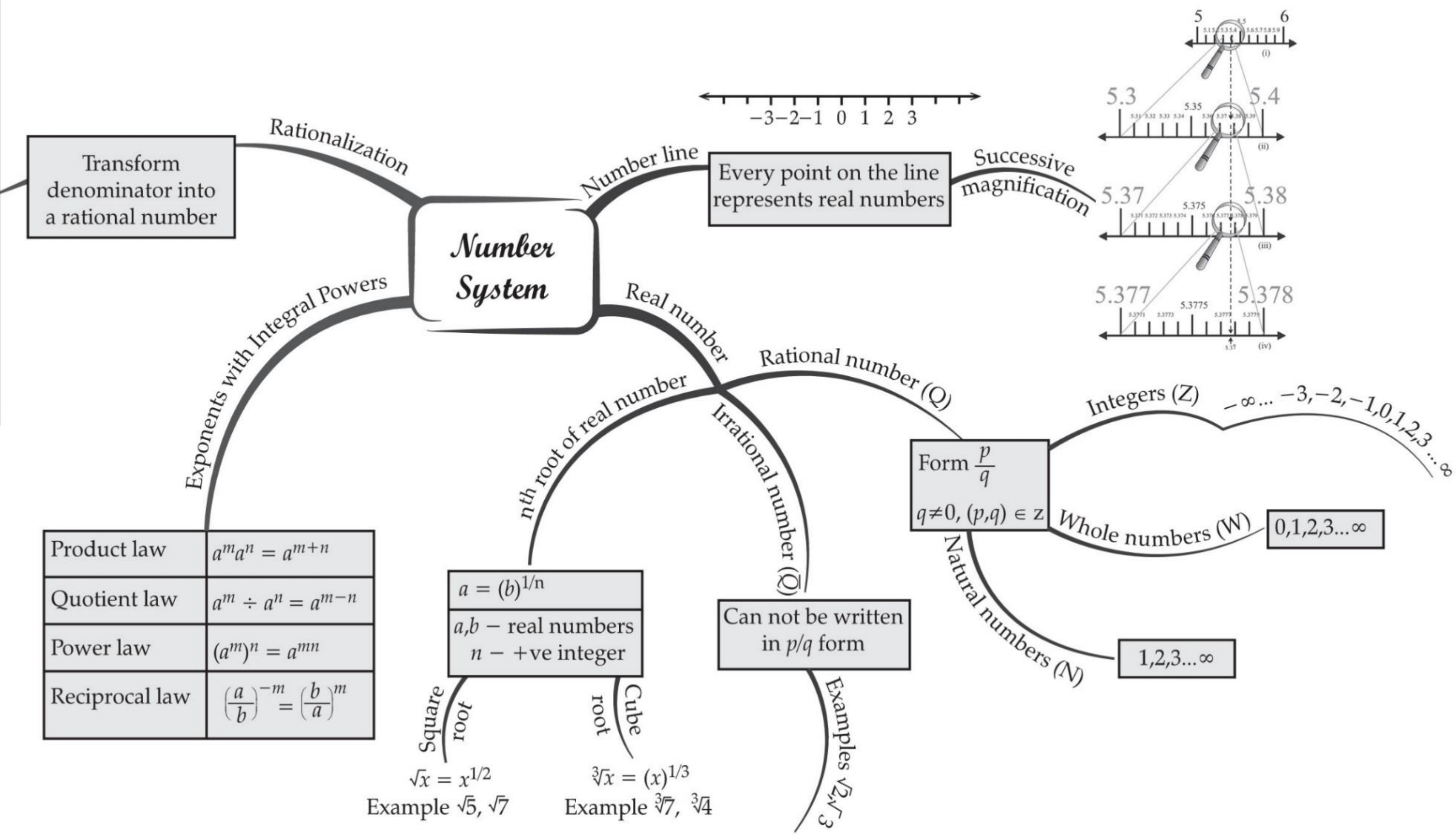
Which is the required rational number.

Reduce the obtained rational number to its simplest form Thus,

X = 172/165

CHAPTER : 1 NUMBER SYSTEM

Term	Rationalising factor
$\frac{1}{\sqrt{r}}$	$\sqrt{r}$
$\frac{1}{\sqrt{r}-s}$	$\sqrt{r}+s$
$\frac{1}{\sqrt{r}+s}$	$\sqrt{r}-s$
$\frac{1}{\sqrt{r}-\sqrt{s}}$	$\sqrt{r}+\sqrt{s}$
$\frac{1}{\sqrt{r}+\sqrt{s}}$	$\sqrt{r}-\sqrt{s}$



Important Questions

Multiple Choice Questions-

Question 1. Can we write 0 in the form of p/q?

a. Yes

b. No

c. Cannot be explained

d. None of the above

Question 2. The three rational numbers between 3 and 4 are:

a. 5/2,6/2,7/2

b. 13/4,14/4,15/4

c. 12/7,13/7,14/7

d.11/4,12/4,13/4

Question 3. In between any two numbers there are:

a. Only one rational number

b. Many rational numbers

c. Infinite rational numbers

d. No rational number

Question 4. Every rational number is:

a. Whole number

b. Natural number

c. Integer

d. Real number

Question 5. √9 is a ___________ number.

a. Rational

b. Irrational

c. Neither rational or irrational

d. None of the above

Question 6. Which of the following is an irrational number?

a. √16

b. √(12/3)

c. √12

d. √100

Question 7. $3\sqrt{6} + 4\sqrt{6}$ is equal to:

a. $6\sqrt{6}$

b. $7\sqrt{6}$

c. $4\sqrt{12}$

d. $7\sqrt{12}$

Question 8. $\sqrt{6} \times \sqrt{27}$ is equal to:

a. $9\sqrt{2}$

b. $3\sqrt{3}$

c. $2\sqrt{2}$

d. $9\sqrt{3}$

Question 9. Which of the following is equal to x3?

a. x^6-x^3

b. $x^6.x^3$

c. x^6/x^3

d. $(x^6)^3$

Question 10. Which of the following are irrational numbers?

a. $\sqrt{23}$

b. $\sqrt{225}$

c. 0.3796

d. 7.478478

Very Short:

1. Simplify: $(\sqrt{5} + \sqrt{2})^2$.
2. Find the value of $\sqrt{(3)^{-2}}$.
3. Identify a rational number among the following numbers:
4. Express 1.8181… in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0.
5. Simplify: $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$
6. Find the value of'

$$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}}$$

7. Find the value of.

$$\frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}}$$

Short Questions:

1. Evaluate: $(\sqrt{5} + \sqrt{2}^2 + (\sqrt{8} - \sqrt{5})^2$
2. Express 23.43 in $\frac{p}{q}$ Form, where p, q are integers and $q \neq 0$.
3. Let 'a' be a non-zero rational number and 'b' be an irrational number. Is 'ab' necessarily an irrational? Justify your answer with example.
4. Let x and y be a rational and irrational numbers. Is x + y necessarily an irrational number? Give an example in support of your answer.
5. Represent $\sqrt{3}$ on the number line.
6. Represent $\sqrt{3.2}$ on the number line.
7. Express 1.32 + 0.35 as a fraction in the simplest form.

Long Questions:

1. If $x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$, then prove that $q^2 - 2px + 9 = 0$.
2. If $a = \frac{1}{3-\sqrt{11}}$ and $b = \frac{1}{a}$, then find $a^2 - b^2$
3. Simplify

$$\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}.$$

4. Prove that:

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{8}+3} = 2.$$

5. Find a and b, if

$$\frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{3}} + \frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = a + \sqrt{15}b$$

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: 0.271 is a terminating decimal and we can express this number as 271/1000 which is of the form p/q , where p and q are integers and q ≠ 0.

Reason: A terminating or non-terminating decimal expansion can be expressed as rational number.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: Every integer is a rational number.

Reason: Every integer 'm' can be expressed in the form m/1 .

Answer Key:

MCQ:

1. (a) Yes
2. (b) 13/4,14/4,15/4
3. (c) Infinite rational numbers
4. (d) Real number
5. (a) Rational
6. (c) √12
7. (b) 7√6
8. (a) 9√2
9. (c) x6/x3
10. (a) √23

Very Short Answer:

1. Here, (√5 + √22 = (√52 + 2√5√2 + (√2)2

   = 5 + 2√10 + 2 = 7 + 2√10

2. $\sqrt{(3)^{-2}} = (3^{-2})^{\frac{1}{2}} = 3^{-2\times\frac{1}{2}} = 3^{-1} = \frac{1}{3}$.

3. O is a rational number.

4. Let x =1.8181… …(i)

100x = 181.8181… …(ii) [multiplying eqn. (i) by 100]

99x = 180 [subtracting (i) from (ii)]

$x = \frac{180}{99}$

Hence, 1.8181 ... $= \frac{180}{99} = \frac{20}{11}$

5. √45 – 3√20 + 4√5 = 3√5 – 6√5 + 4√5 = √5.

6. $$\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}} = \frac{1-\frac{1}{0.1}}{\frac{8}{3}\times\frac{27}{8}+(-3)} = \frac{1-10}{9-3} = \frac{-9}{6} = -\frac{3}{2}$$

7.

$$\frac{4}{(216)^{\frac{-2}{3}}}-\frac{1}{(256)^{\frac{-3}{4}}} = 4\times(216)^{\frac{2}{3}}-(256)^{\frac{3}{4}} = 4\times(6\times6\times6)^{\frac{2}{3}}-(4\times4\times4\times4)^{\frac{3}{4}}$$
$$= 4\times6^{3\times\frac{2}{3}}-4^{4\times\frac{3}{4}} = 4\times6^2-4^3$$
$$= 4\times36-64 = 144-64 = 80$$

Short Answer:

Ans: 1. (√5 + √2)2 + (√8 – √52 = 5 + 2 + 2√10 + 8 + 5 – 2√40
= 20 + 2√10 – 4√10 = 20 – 2√10

Ans: 2. Let x = $23.\overline{43}$

or x = 23.4343… ….(i)

100x = 2343.4343… …(ii) [Multiplying eqn. (i) by 100]

99x = 2320 [Subtracting (i) from (ii)

$\Rightarrow x = \frac{2320}{99}$

Hence, $23.\overline{43} = \frac{2320}{99}$

Ans: 3. Yes, 'ab' is necessarily an irrational.

For example, let a = 2 (a rational number) and b = √2 (an irrational number)

If possible let ab = 2√2 is a rational number.

Now $\frac{ab}{a} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ is a rational number.

[∵ The quotient of two non-zero rational number is a rational]

But this contradicts the fact that √2 is an irrational number.

Thus, our supposition is wrong.

Hence, ab is an irrational number.

Ans: 4. Yes, x + y is necessarily an irrational number.

For example, let x = 3 (a rational number) and y = √5 (an irrational number)

If possible, let x + y = 3 + √5 be a rational number.

Consider $\frac{p}{q}$ = 3 + √5, where p, q ∈ Z and q ≠ 0.

Squaring both sides, we have

$$\frac{p^2}{q^2} = 9 + 5 + 6\sqrt{5} \quad \Rightarrow \quad \frac{p^2}{q^2} = 14 + 6\sqrt{5}$$

$$\Rightarrow \frac{p^2}{q^2} - 14 = 6\sqrt{5} \quad \Rightarrow \quad \frac{p^2 - 14q^2}{6q^2} = \sqrt{5}$$

$$\because \frac{p}{q} \text{ is a rational} \quad \Rightarrow \quad \frac{p^2 - 14q^2}{6q^2} \text{ is a rational}$$

$\because \frac{p}{q}$ is a rational

⇒ √5 is a rational

But this contradicts the fact that √5 is an irrational number.

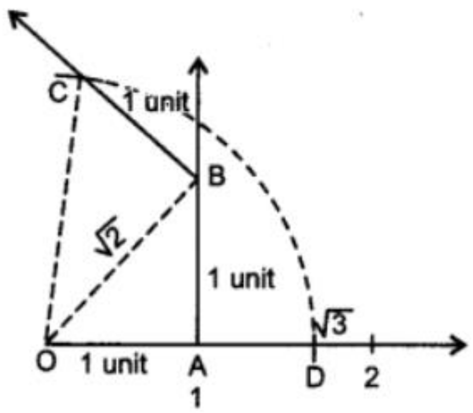
Thus, our supposition is wrong.

Hence, x + y is an irrational number.

Ans: 5.

$$\text{Here,} \quad \sqrt{3} = \sqrt{1+2} = \sqrt{(1)^2 + (\sqrt{2})^2}$$

$$\text{And,} \quad \sqrt{2} = \sqrt{1+1} = \sqrt{(1)^2 + (1)^2}$$



On the number line, take OA = 1 unit. Draw AB = 1 unit perpendicular to OA. Join OB.

Again, on OB, draw BC = 1 unit perpendicular to OB. Join OC.

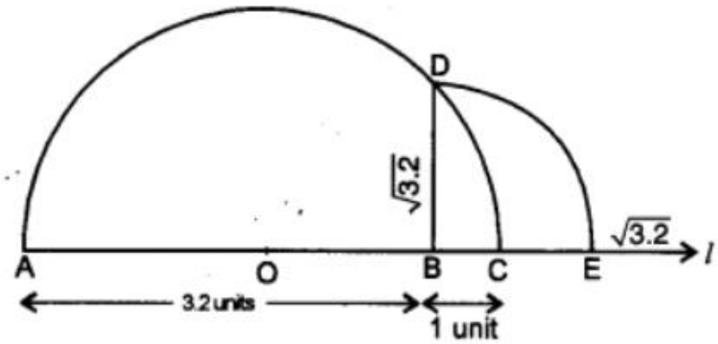
By Pythagoras Theorem, we obtain OC = √3. Using

compasses, with Centre O and radius OC, draw an arc, which intersects the number line at point

D. Thus, OD = √3 and D corresponds to √3.

Ans: 6. First of all draw a line of length 3.2 units such that AB = 3.2 units. Now, from point B, mark a distance of 1 unit. Let this point be ‘C’. Let ‘O’ be the mid-point of the distance AC. Now, draw a semicircle with Centre ‘O’ and radius OC. Let us draw a line perpendicular to AC passing through the point ‘B’ and intersecting the semicircle at point ‘D’.

∴ The distance BD = √3.2



Now, to represent √3.2 on the number line. Let us take the line BC as number line and point ‘B’ as zero, point ‘C’ as ‘1’ and so on. Draw an arc with Centre B and radius BD, which intersects the number line at point ‘E’.

Then, the point ‘E’ represents √3.2.

Ans: 7. Let. x = 1.32 = 1.3222…. (i)

Multiplying eq. (i) by 10, we have

10x = 13.222…

Again, multiplying eq. (i) by 100, we have

100x = 132.222… …(iii)

Subtracting eq. (ii) from (iii), we have

100x – 10x = (132.222…) – (13.222…)

90x = 119

$\Rightarrow x = \frac{119}{90}$

Again, y = 0.35 = 0.353535……

Multiply (iv) by 100, we have …(iv)

100y = 35.353535… (v)

Subtracting (iv) from (u), we have

100y – y = (35.353535…) – (0.353535…)

99y = 35

$y = \frac{35}{99}$

Now, $1.3\overline{2} + 0.\overline{35} = x + y = \frac{119}{90} + \frac{35}{99} = \frac{1309 + 350}{990} = \frac{1659}{990} = \frac{553}{330}$

Long Answer:

Ans: 1.

$$x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}} = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}} \times \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$$

$$= \frac{\left(\sqrt{p+q}+\sqrt{p-q}\right)^2}{\left(\sqrt{p+q}\right)^2-\left(\sqrt{p-q}\right)^2} = \frac{p+q+p-q+2\times\sqrt{p+q}\times\sqrt{p-q}}{(p+q)-(p-q)}$$

$$= \frac{2p+2\sqrt{p^2-q^2}}{2q} = \frac{p+\sqrt{p^2-q^2}}{q}$$

$$\Rightarrow \quad qx = p+\sqrt{p^2-q^2}$$

$$\Rightarrow qx - p = \sqrt{p^2-q^2}$$

Squaring both sides, we have

$\Rightarrow q^2x^2 + p^2 - 2pqx = p^2 - q^2$

$\Rightarrow q^2x^2 - 2pqx + q^2 = 0$

$\Rightarrow q\ (q^2 - 2px + q) = 0$

$\Rightarrow qx^2 - 2px + q = 0\ (\because q \neq 0)$

Ans: 2

Here, $a = \frac{1}{3-\sqrt{11}} \times \frac{3+\sqrt{11}}{3+\sqrt{11}} = \frac{3+\sqrt{11}}{9-11} = \frac{3+\sqrt{11}}{-2}$

$b = \frac{1}{a} = 3-\sqrt{11}$

Now, $a^2 - b^2 = (a+b)(a-b)$

$$= \left(\frac{3+\sqrt{11}}{-2}+3-\sqrt{11}\right)\left(\frac{3+\sqrt{11}}{-2}-3+\sqrt{11}\right)$$

$$= \left(\frac{-3-\sqrt{11}+6-2\sqrt{11}}{2}\right)\left(\frac{-3-\sqrt{11}-6+2\sqrt{11}}{2}\right)$$

$$= \left(\frac{3-3\sqrt{11}}{2}\right)\left(\frac{-9+\sqrt{11}}{2}\right) = \frac{-27+3\sqrt{11}+27\sqrt{11}-33}{4}$$

$$= \frac{-60+30\sqrt{11}}{4} = \frac{-30+15\sqrt{11}}{2} = \frac{1}{2}(15\sqrt{11}-30)$$

Ans: 3

$$\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+2}$$

$$=\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}\times\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+2}\times\frac{\sqrt{6}-2}{\sqrt{6}-2}$$

$$=\frac{3\sqrt{12}+3\sqrt{6}}{(\sqrt{6})^2-(\sqrt{3})^2}-\frac{4\sqrt{18}+4\sqrt{6}}{(\sqrt{6})^2-(\sqrt{2})^2}+\frac{2\sqrt{18}-4\sqrt{3}}{(\sqrt{6})^2-(2)^2}$$

$$=\frac{6\sqrt{3}+3\sqrt{6}}{6-3}-\frac{12\sqrt{2}+4\sqrt{6}}{6-2}+\frac{6\sqrt{2}-4\sqrt{3}}{6-4}$$

$$=\frac{6\sqrt{3}+3\sqrt{6}}{3}-\frac{12\sqrt{2}+4\sqrt{6}}{4}+\frac{6\sqrt{2}-4\sqrt{3}}{2}$$

$$=\frac{24\sqrt{3}+12\sqrt{6}-36\sqrt{2}-12\sqrt{6}+36\sqrt{2}-24\sqrt{3}}{12}=\frac{0}{12}=0.$$

Ans: 4.

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{8}+3}$$

$$=\frac{1}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}\times\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}}+\ldots+\frac{1}{\sqrt{8}+3}\times\frac{\sqrt{8}-3}{\sqrt{8}-3}$$

$$=\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{\sqrt{3}-\sqrt{4}}{-1}+\ldots+\frac{\sqrt{8}-3}{-1}$$

$$=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}-\ldots-\sqrt{8}+3$$

$$=-1+3=2$$

Ans: 5.

Here, $\frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{3}}+\frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}}=a+\sqrt{15}b$

$$\frac{(2\sqrt{5}+\sqrt{3})^2+(2\sqrt{5}-\sqrt{3})^2}{(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})}=a+\sqrt{15}b$$

$$\frac{20+3+4\sqrt{15}+20+3-4\sqrt{15}}{20-3}=a+\sqrt{15}b$$

$$\frac{46}{17}=a+\sqrt{15}b$$

Comparing rational and irrational parts, we have

$$a=\frac{46}{17} \text{ and } b=0$$

Assertion and Reason Answers-

1. c) Assertion is correct statement but reason is wrong statement.

2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.