MATHEMATICS

Chapter 1: Integers

Integers

1. Set of positive numbers, 0 and negative numbers is called integers. The set of integers is denoted by I or Z.
2. Natural numbers are contained in the whole numbers.
3. Whole numbers are contained in the integers.
4. Negative numbers are placed on left side of '0' on the horizontal number line. 0 is less than every positive integer and greater than every negative integer.
5. Absolute value of a number is only its numerical value without considering the sign into account.

Properties of integers

i. Closure property is satisfied with respect to addition, subtraction and multiplication of the integers.

For a, b $\in$ I, we have:

$a + b \in I$

$a - b \in I$

$a \times b \in I$

ii. Commutative property is satisfied with respect to addition and multiplication of the integers.

If a, b $\in$ I, then

$a + b = b + a$

$a \times b = b \times a$

iii. Associative property is satisfied with respect to addition and multiplication of the integers.

If a, b, c $\in$ I, then

$a + (b + c) = (a + b) + c = c + (b + a)$

$a \times (b \times c) = (a \times b) \times c = c \times (b \times a)$

iv. Distributive property: Multiplication is distributive over addition and subtraction of the integers.

v. a and -a are additive inverses of each other.

vi. 0 is the additive identity for integers.

vii. 1 is the multiplicative identity for integers.

viii. Division by 0 is not defined.

Adding two positive integers results in positive integers, whereas adding two negative integers will result in the sum with a negative sign. But, the addition of two different signed integers will result in subtraction only and the sign of the result will be the same as the larger number has. See a few examples below:

2+2 = 4

2 + (-2) = 0

-2 + (-2) = -4

-2 – (-2) = 0

Name	Addition	Substraction
Closure	a+b∈z	a-b∈z
Commutative	a+b = b+a	a-b ≠ b-a
Associative	(a+b) + c = a+(b+c)	(a-b) - c ≠ a-(b-c)
Distributive	a x (b+c) = ab + ac	a x (b-c) = ab + ac

Addition and subtraction

Addition and subtraction are two primary arithmetic operations in Maths. Besides these two operations, multiplication and division are also two primary operations that we learn in basic Maths.

The addition represents the values added to the existing value. For example, a basket has two balls, and if we add more than 2 balls to it, there will be four balls in total. Similarly, if there are four balls in a basket and if we take out two balls out of it, then the basket is left with only two balls, which shows subtraction.

Addition and subtraction are not only used for integers but also rational numbers and irrational numbers. Therefore, both the operations are applicable for all real numbers and complex numbers. Also, the addition and subtraction algebraic expressions are done based on the same rules while performing algebraic operations.

Rules to Add and Subtract

Integers are a special group of numbers that are positive, negative and zero, which are not fractions. Rules for addition and subtraction are the same for all.

Negative Sign and Positive Sign

The integers which we add or subtract could be positive or negative. Hence, it is necessary to know the rules for positive and negative symbols.

Positive sign/symbol: (+)

Negative sign/symbol: (-)

Addition of Integers

The three main possibilities in the addition of integers are:

- Addition between two positive numbers
- Addition between two negative numbers
- Addition between a positive number and a negative number

Type of Numbers	Operation	Result	Example
Positive + Positive	Add	Positive (+)	10 + 15 = 25
Negative + Negative	Add	Negative (-)	(-10) + (-15) = -25
Positive + Negative*	Subtract	Positive (+)	(-10) + 15 =5
Negative + Positive*	Subtract	Negative (-)	10 + (-15)= -5

Whenever a positive number and a negative number are added, the sign of the greater number will decide the operation and sign of the result. In the above example 10 + (-15) = -5 and (-10) + 15 =5; here, without sign 15 is greater than 10 hence, numbers will be subtracted and the answer will give the sign of the greater number.

We know that the multiplication of a negative sign and a positive sign will result in a negative sign, therefore if we write 10 + (-5), it means the '+' sign here is multiplied by '-' inside the bracket. Therefore, the result becomes 10 – 5 = 5.

Alternatively, to find the sum of a positive and a negative integer, take the absolute value ("absolute value" means to remove any negative sign of a number, and make the number positive) of each integer and then subtract these values. Take the above example, 10 + (-15); absolute value of 10 is 10 and -15 is 15.

⇒ 10 – 15 = -5

Thus, we can conclude the above table as follow:

- Adding two positive integers results in positive integer
- Adding two negative integers results in a sum of integers with a negative sign.
- The addition of a positive and a negative integer gives either a positive or negative-sum depending on the value of the given numbers.

Note: The sum of an integer and its opposite is always zero. (For example, -5 + 5= 0)

Subtraction of Integers

Like in addition, the subtraction of integers also has three possibilities. They are:

- Subtraction between two positive numbers
- Subtraction between two negative numbers

- Subtraction between a positive number and a negative number

For ease of calculation, we need to renovate subtraction problems the addition problems. There are two steps to perform this and are given below.

Convert the subtraction sign into an addition sign.

After converting the sign, take the inverse of the number which comes after the sign.

Once the transformation is done, follow the rules of addition given above.

For example, finding the value of (-5) – (7)

Step 1: Change the subtraction sign into an addition sign

⇒ (-5) + (7)

Step 2: Take the inverse of the number which comes after the sign

⇒ –5 + (-7) (opposite of 7 is -7)

⇒ –5 + (-7) = -12 [Add and put the sign of greater number]

Additive Identity & Additive Inverse

Additive Identity

For every integer a, a + 0 = 0 + a = a here 0 is Additive Identity, since adding 0 to a number leaves it unchanged.

Example: For an integer 2, 2 + 0 = 0 + 2 = 2.

Additive inverse

For every integer a, a + (−a) = 0 Here, −a is additive inverse of a and a is the additive inverse of-a.

Example: For an integer 2, (– 2) is additive inverse and for (– 2), additive inverse is 2. [Since + 2 – 2 = 0]

Properties of Multiplication of Integers

Closure under Multiplication

For every integer a and b, a × b = Integer

Commutative Property of Multiplication

For every integer a and b, a × b = b × a

Multiplication by Zero

For every integer a, a × 0 = 0 × a = 0

Multiplicative Identity

For every integer a, a × 1 = 1 × a = a. Here 1 is the multiplicative identity for integers.

Associative property of Multiplication

For every integer a, b and c, (a × b) × c = a × (b × c)

Distributive Property of Integers

Under addition and multiplication, integers show the distributive property.

i.e., For every integer a, b and c, $a \times (b + c) = a \times b + a \times c$

These properties make calculations easier.

Multiplication is basically the repeated addition of numbers. For example, if we say, 2 multiplied by 3, it means 2 is added to itself three times.

2 x 3 = 2 + 2 + 2 = 6

Therefore multiplication of integers is the repeated addition as:

$$a \times n = \underbrace{a + a + a \ldots . . a}_{n\ times}$$

Where a and n are both integers.

Division of Integers

When a positive integer is divided by a positive integer, the quotient obtained is a positive integer.

Example: (+6) ÷ (+3) = +2

When a negative integer is divided by a negative integer, the quotient obtained is a positive integer.

Example: (-6) ÷ (-3) = +2

When a positive integer is divided by a negative integer or negative integer is divided by a positive integer, the quotient obtained is a negative integer.

Example: (-6) ÷ (+3) =−2 and Example: (+6) ÷ (-3) = −2

Introduction to Zero

Integers

Integers are the collection of numbers which is formed by whole numbers and their negatives.

The set of Integers is denoted by Z or I. I = { ..., -4, -3, -2, -1, 0, 1, 2, 3, 4,... }

Operations on integers

Addition of integers

i. To add two positive numbers, we add their values and give positive sign to the result.

   Ex: (+5) + (+2) = +7

ii. To add two negative numbers, we add their values and give negative sign to the result.

   Ex: (-12) + (-3) = -15

iii. One positive and one negative number are added by subtracting them and giving the

sign of the greater number to result. The greater number is decided by ignoring the signs of the integers.

Ex: (+24) + (-3) = +21

(-32) + (+8) = -24

Subtraction of integers

While subtracting integers, Minuend remains the same and sign of the subtrahend will be changed and then added.

Subtraction is nothing but adding the minuend and subtrahend by changing the sign of the subtrahend.

Ex: (+15) – (+12)

= (+15) + (-12)

(Keep minuend same, change the sign of subtrahend and then add)

=+3

(One +ve and one -ve. Subtract and then give bigger number sign)

Multiplication of integers

To multiply two integers, first signs are multiplied and then the numerals are multiplied.

The multiplication of the signs is as follows:

(+) × (+) = +

(-) × (-) = +

(+) × (-) = -

(-) × (+) = -

Division of integers

To divide two integers, first signs will be divided and then the numerals are divided.

The division of the signs is as follows:

(+) ÷ (+) = +

(-) ÷ (-) = +

(+) ÷ (-) = -

(-) ÷ (+) = -

6. In an exponential number, base is the number which is repeatedly multiplied. Exponent is the number which tells how many times the base has to be multiplied. In x^7, x is called base and 7 is called exponent or index or power.
7. If the exponent of a negative integer is odd then the value is negative.

   $(-5)^3 = (-5) \times (-5) \times (-5) = -125$
8. If the exponent of a negative integer is even then the value is positive.
9. Using BODMAS rule, we simplify the numerical expressions.

   The order of removing the brackets of an expression is as follows:

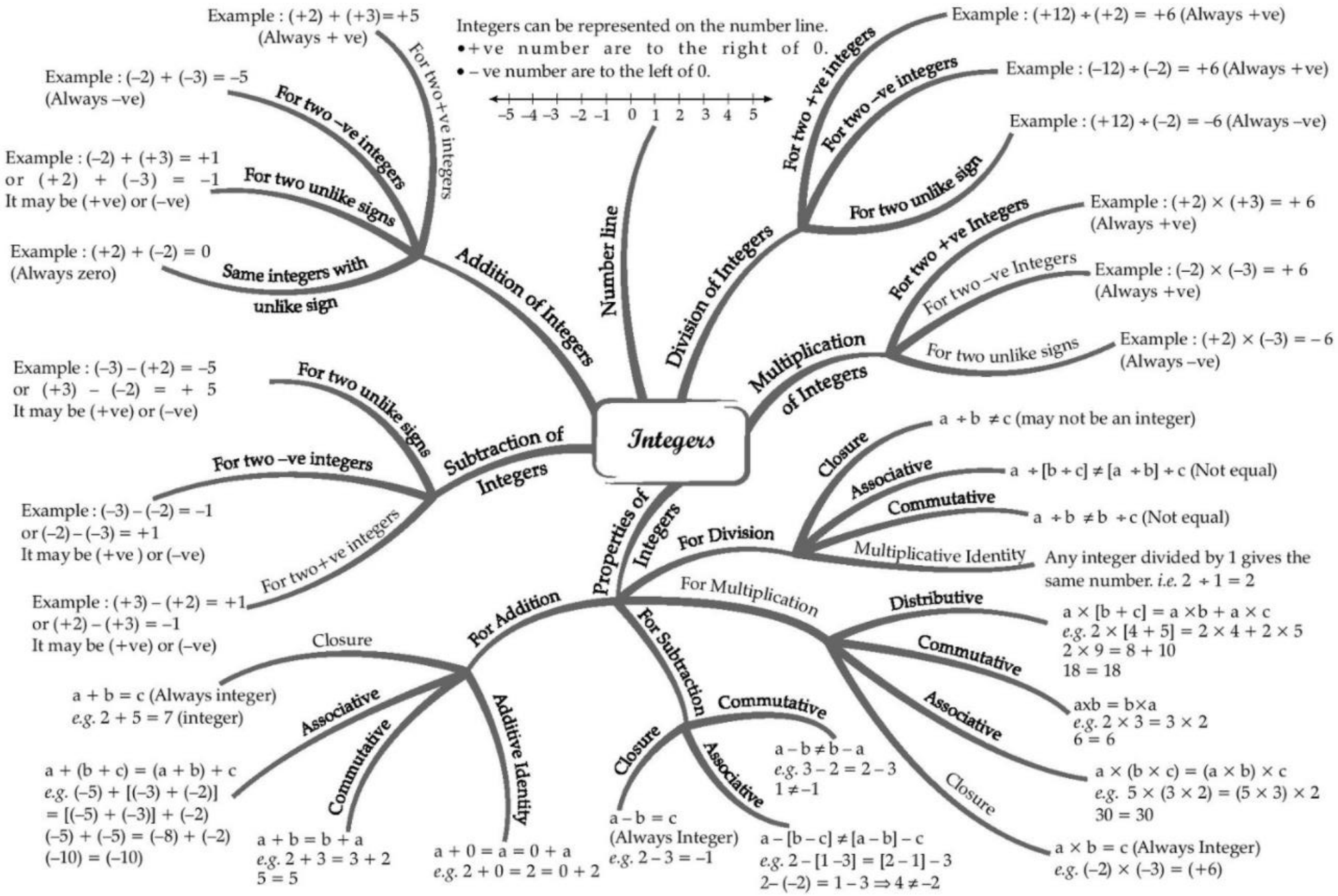
   () Parethesis, {} Flower or curly brackets and [] Square brackets First we remove () then

{} Lastly [].

If a bracket is preceded by a negative term, then on removing bracket the signs of the terms inside the bracket will be changed.

If a bracket is preceded by a positive term, then on removing the bracket the signs of the terms inside the bracket will not change.

CHAPTER - 1 INTEGERS



Important Questions

Multiple Choice Questions :

Question 1. −67 × (−1) = ?

(a) -1

(b) -67

(c) 67

(d) 1

Question 2. Find 4 x (- 8)

(a) – 32

(b) 32

(c) None of these

Question 3. With respect to which of the following operations is closure property satisfied by the set of integers?

(a) +,×

(b) +,÷,×

(c) +,×,−

(d) +,−,÷

Question 4. On a number line, when we subtract a positive integer, we

(a) move to the right

(b) move to the left

(c) do not move at all

(d) none of these

Question 5. 10 × (−3) = ?

(a) 7

(b) 30

(c) -30

(d) None of these

Question 6. What is the absolute value of |-239|?

(a) 0

(b) 239

(c) -239

(d) 1

Question 7. Additive inverse of 10 is :

(a) 0

(b) 10

(c) -10

(d) None of these

Question 8. What will be the sign of the product if we multiply together 8 negative integers ?

(a) Negative

(b) Positive

(c) None of these

Question 9. 2 × 4 = ?

(a) 8

(b) -8

(c) 3

(d) 6

Question 10. Find 0 x 7

(a) 7

(b) 0

(c) None of these

Question 11. 6 × (−15 + 10) = _____

(a) 30

(b) -21

(c) -30

(d) 21

Question 12. If a, b, c are 3 integers then, a + (b + c)=

(a) a + b + c

(b) (a + b) + c

(c) (a + c) + b

(d) None of these

Question 13. Where are the negative numbers located on a horizontal number line?

(a) On the right of 0

(b) On the left of 0

(c) Above 0

(d) Below 0

Question 14. What is the value of $124 \times 4-3 + 118 \div 2$?

(a) 552

(b) 496

(c) 553

(d) −553

Question 15. Evaluate of $-50 \div 5$

(a) -10

(b) 10

(c) None of these

Fill in the blanks :

Fill in the blanks using < or >.

(a) -3 …… -4

(b) 6 ……. -20

(c) -8 …… -2

(d) 5 …… -7

Very Short Questions :

1. Solve the following:

   (-8) × (-5) + (-6)

2. Solve the following:

   [(-6) × (-3)] + (-4)

3. Solve the following:

   (-10) × [(-13) + (-10)]

4. Solve the following:

   (-5) × [(-6) + 5]

5. Starting from (-7) × 4, find (-7) × (-3)

6. Using number line, find:

   (i) 3 × (-5)

7. Using number line, find:

   (i) 8 × (-2)

8. Write five pair of integers (m, n) such that m ÷ n = -3. One of such pair is (-6, 2).

Short Questions :

1. Solve the following:

(-15) × 8 + (-15) × 4

2. Solve the following:

[32 + 2 × 17 + (-6)] ÷ 15

3. The sum of two integers is 116. If one of them is -79, find the other integers.

4. If a = -35, b = 10 cm and c = -5, verify that:

(i) a + (b + c) = (a + b) + c

(ii) a × (b + c) = a × b + a × c

5. Write down a pair of integers whose

(i) sum is -5

(ii) difference is -7

(iii) difference is -1

(iv) sum is 0

Long Questions :

1. You have ₹ 500 in your saving account at the beginning of the month. The record below shows all of your transactions during the month. How much money is in your account after these transactions?

Cheque No.	Date	Transactions description	Payment	Deposit
384102	4/9	Jal Board	₹ 120	₹ 200
275146	12/9	Deposit		
384103	22/9	LIC India	₹ 240	₹ 150
801351	29/9	Deposit		

2. The given table shows the freezing points in °F of different gases at sea level. Convert each of these into °C to the nearest integral value using the relations and complete the table

$$C = \frac{5}{9}[F - 32]$$

Gas	Freezing point at sea level (°F)	Freezing point at sea level (°C)
Hydrogen	–435	
Krypton	–251	
Oxygen	–369	

3. Taking today as zero on the number line, if the day before yesterday is 17 January, what is the date on 3 days after tomorrow?

Assertion and Reason Questions:

1) Assertion: The integers on the number line forms an infinite sequence.

Reason: A list of numbers following a definite rule which goes on forever is called an infinite sequence.

a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

c.) assertion is true but the reason is false.

d.) both assertion and reason are false.

2) Assertion: Every integer is a rational number.

Reason: An integer is a number with no decimal or fractional part, from the set of negative and positive numbers, including zero.

a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

c.) assertion is true but the reason is false.

d.) both assertion and reason are false.

ANSWER KEY -

Multiple Choice questions :

1. (c) 67
2. (a) – 32

   Product of 2 numbers of opposite signs is negative.
3. (c) +,×,−
4. (b) move to the left
5. (c) -30
6. (b) 239
7. (c) -10

   If a is an integer then (- a) is its additive inverse.
8. (b) Positive

   Since 8 is even so the product of 8 negative integers is positive.

9. (a) 8

10. (b) 0

Zero multiplied by any integer is zero.

11. (c) -30

12. (b) (a + b) + c

The addition of integer is associative.

13. (b) On the left of 0

14. (a) 552

15. (a) -10

Fill in the blanks :

(a) -3 $\geq$ -4

(b) 6 $\geq$ -20

(c) -8 $\leq$ -2

(d) 5 $\geq$ -7

Very Short Answer :

1. (-8) × (-5) + (-6)

= (-) × (-) × [8 × 5] + (-6)

= 40 – 6

= 34

2. [(-6) × (-3)] + (-4)

= (-) × (-) × [6 × 3] + (-4)

= 18 – 4

= 14

3. (-10) × [(-13) + (-10)]

= (-10) × (-23)

= (-) × (-) × [10 × 23]

= 230

4. (-5) × [(-6) + 5]

= (-5) × (-1)

= (-) × (-) × 5 × 1

= 5

5. (-7) × 4 = -28

(-7) × 3 = -21 = [-28 + 7]

(-7) × 2 – -14 = [-21 + 7]

(-7) × 1 = -7 = [-14 + 7]

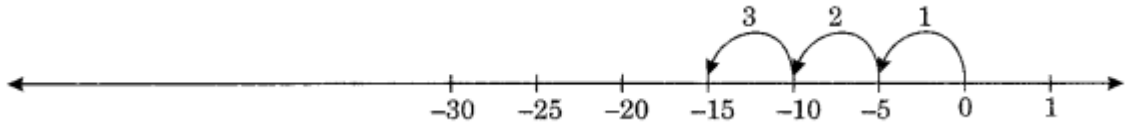
(-7) × 0 = 0 = [-7 + 7]

(-7) × (-1) = 7 = [0 + 7]

(-7) × (-2) = 14 = [7 + 7]

(-7) × (-3) = 21 = [14 + 7]

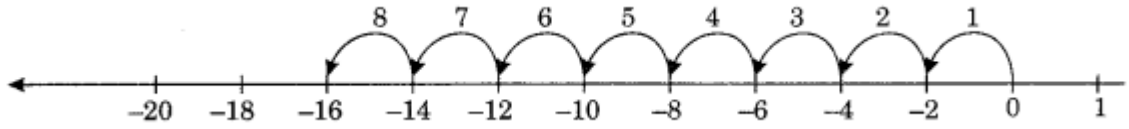
6. (i) 3 × (-5)



From the number line, we have

(-5) + (-5) + (-5) = 3 × (-5) = -15

7. (i) 8 × (-2)



From the number line, we have

(-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) = 8 × (-2) = -1

8. (i) (-3, 1) = (-3) ÷ 1 = -3

(ii) (9, -3) = 9 ÷ (-3) = -3

(iii) (6, -2) = 6 ÷ (-2) = -3

(iv) (-24, 8) = (-24) ÷ 8 = -3

(v) (18, -6) = 18 ÷ (-6) = -3

Short Answer :

1. (i) (-15) × 8 + (-15) × 4

= (-15) × [8 + 4]

= (-15) × 12

= -18

2. (ii) [32 + 2 × 17 + (-6)] ÷ 15

= [32 + 34 – 6] ÷ 15

= [66 – 6] ÷ 15

= 60 ÷ 15

= 4

3. Sum of two integers = 116

One integer = -79

Other integer = Sum of integer – One of integer = 116 – (-79) = 116 + 79 = 195

4. (i) Given that a = -35, b = 10, c = -5

LHS = a + (b + c) = (-35) + [10 + (-5)] = (-35) + 5 = -30

RHS = (a + b) + c = [(-35) + 10] + (-5) = (-25) + (-5) = -(25 + 5) = -30

LHS = RHS

Hence, verified

(ii) a × (b + c) = a × b + a × c

LHS = a × (b + c) = (-35) × [10 + (-5)] = (-35) × 5 = -175

RHS = a × b + a × c = (-35) × 10 + (-35) × (-5) = -350 + (-) × (-) × (35 × 5) = -350 + 175 = -175

LHS = RHS

Hence, verified.

5. (i) (-2) + (-3) = -5

Hence, the required pair of integers = (-2, -3)

(ii) -10 – (-3) = -10 + 3 = -7

Hence, the required pair of integers = (-10, -3)

(iii) (-3) – (-2) = -1

Hence, the required pair of integers = (-3, -2)

(iv) (-4) + (4) = 0

Hence, the required pair of integers = (-4, 4)

Long Answer :

1. Amount in the beginning of the month in the account = ₹ 500

Amount deposited in the account for Jal Board = ₹ 200

Amount paid to Jal Board = ₹ 120

Amount left in the account after the above transactions = ₹ (500 + 200 – 120) = ₹ (700 – 120) = ₹ 580

Amount deposited for LIC India = ₹ 150

Amount paid to LIC India = ₹ 240

Amount left after this transactions = ₹ (580 + 150 – 240) = ₹ (730 – 240) = ₹ 490

2. Freezing point of Hydrogen = -435°F

$C = \frac{5}{9}[-435 - 32]$

$= \frac{5}{9}[-467]$

$= 5 \times (-51.9)$

= 259.5°C or 259°C

For Krypton, freezing point = -251°F

$C = \frac{5}{9}[-251 - 32]$

$= \frac{5}{9}[-283]$

$= 5 \times (-31.4)]$

= -157°C

For Oxygen, freezing point = -369°F

$C = \frac{5}{9}[-369 - 32]$

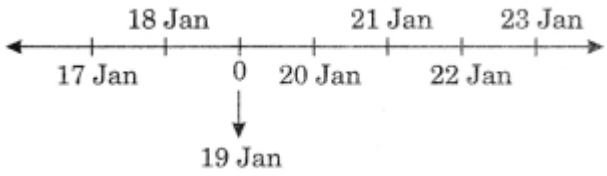
$= \frac{5}{9}[-401]$

$= 5 \times (44.56)$

= 222.80° C or 223°C

Hence, the required freezing points at sea level in °C for Hydrogen = -259°C, Krypton = -157°C, Oxygen = -223°C.

3.



The date before yesterday = 17 January

The date of yesterday = 17 + 1 = 18 January

Today's date = 18 + 1 = 19 January

Tomorrow's date = 19 + 1 = 20 January

Date on 3 days after tomorrow = (20 + 3) = 23rd January.

Assertion and Reason Answers:

1) a) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

2) b) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.