MATHEMATICS

Chapter 7: Congruence of Triangle

Congruence of Triangles

1. Two figures having exactly same shape and size are said to be congruent.
2. Two plane figures are congruent, if each one when superimposed on the other, covers the other exactly.
3. Congruence of some geometrical figures:
   i. Two line segments are congruent if they have the same length.
   ii. Two angles are congruent if they have the same measure.
   iii. Two squares are congruent if they have the same side length.
   iv. Two rectangles are congruent if they have the same length and same breadth.
   v. Two circles are congruent if they have the same radius.
4. Two triangles are said to be congruent, if pairs of corresponding sides and the corresponding angles are equal.

   If ΔABC and ΔPQR are congruent, then we write it as $\Delta ABC \cong \Delta PQR$.
5. The congruency of ΔABC and ΔPQR under the matching $\Delta ABC \leftrightarrow \Delta PQR$ is written as $\Delta ABC \cong \Delta PQR$.

   This means the vertices A, B, C are matched with the vertices P, Q, R respectively.
6. SSS congruence condition: If three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.
7. SAS congruence condition: If two sides and the included angle of one triangle are respectively equal to the two sides and the included angle of another triangle, then the two triangles are congruent.
8. ASA congruence condition: If two angles and the included side of one triangle are respectively equal to the two angles and the included side of another triangle, then the two triangles are congruent.
9. RHS congruence condition: If the hypotenuse and a side of one right angled triangle are equal to the hypotenuse and a side of another right-angled triangle, then two triangles are congruent.
10. Two congruent figures are equal in area but two figures having the same area need not be congruent.

Congruent Figures

Congruent figures are exactly equal in size and shape.

In Geometry, congruence is a term used to define two objects that have same dimensions and shape. Moreover, if one shape can be exactly like the other on turning, flipping and/or sliding it, then the two shapes are said to be congruent to each other. So two congruent figures drawn on a piece of paper can be cut out and placed over one another to match up perfectly. When we say a figure A is congruent to a figure B, symbolically it can be written as figure A ≅ figure B. In this article, we will discuss congruence of plane figures and line segments and congruent angles.

Congruence of Plane Figures

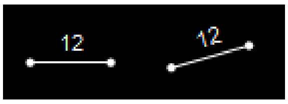
You are given two plane figures. Using the method of superposition, here are the steps to find whether they are congruent to each other or not:

Congruent Figures

- Trace a copy of figure 1 using a tracing paper.
- Cut it out of the paper and place it over figure 2.
- You may slide or move or even turn over the paper. If they cover each other completely, they are congruent. We write figure 1 ≅ figure 2.

Congruence Among Line Segments

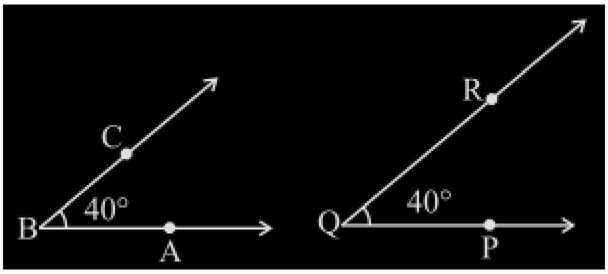
Two line segments are said to be congruent to each other if they have equal lengths. They may or may not align at the same angle with an axis or position in plane. To find whether one line segment is congruent to the other or not, we use the same method of superposition as discussed above. Thus if the two ends of the two line segments lie on one another, they are congruent.



Congruent Line Segments

Congruent Angles

If two angles have the same measure in degrees, they are congruent angles. The angles may or may not lie in the same position or orientation on plane. The method of superposition (as described above) can be used to check if given angles are congruent angles or not.



Congruent Angles

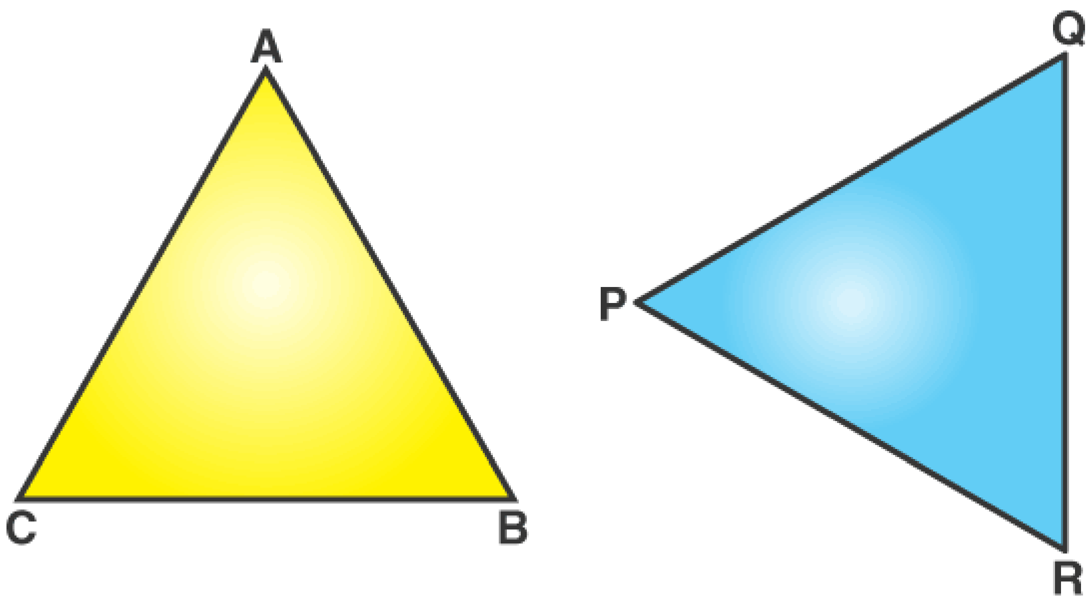
Thus, ∠PQR ≅ ∠ABC because the two angles measure 40 degrees. It is to be noted here that the length of BC is not equal to the length of QR. But congruence of angles does not depend on the length of the arms of the angles; it depends on the measures of the angles only. Thus, ∠PQR and ∠ABC are congruent angles.

Congruent Triangles

If all the sides and angles of a triangle are equal to the corresponding sides and angles of another triangle, then both the triangles are said to be congruent.

The meaning of congruent in Maths is addressed to those figures and shapes that can be repositioned or flipped to coincide with the other shapes. These shapes can be reflected to coincide with similar shapes.

Two shapes are congruent if they have the same shape and size. We can also say if two shapes are congruent, then the mirror image of one shape is same as the other.



In the above figure, Δ ABC and Δ PQR are congruent triangles. This means,

Vertices: A and P, B and Q, and C and R are the same.

Sides: AB=PQ, QR= BC and AC=PR;

Angles: ∠A = ∠P, ∠B = ∠Q, and ∠C = ∠R.

Congruent triangles are triangles having corresponding sides and angles to be equal. Congruence is denoted by the symbol "≅". They have the same area and the same perimeter.

CPCT Full Form

CPCT is the term we come across when we learn about the congruent triangle. CPCT means "Corresponding Parts of Congruent Triangles". As we know that the corresponding parts of congruent triangles are equal. While dealing with the concepts related to triangles and solving questions, we often make use of the abbreviation cpct in short words instead of full form.

CPCT Rules in Maths

The full form of CPCT is Corresponding parts of Congruent triangles. Congruence can be predicted without actually measuring the sides and angles of a triangle. Different rules of congruency are as follows.

SSS (Side-Side-Side)

SAS (Side-Angle-Side)

ASA (Angle-Side-Angle)

AAS (Angle-Angle-Side)

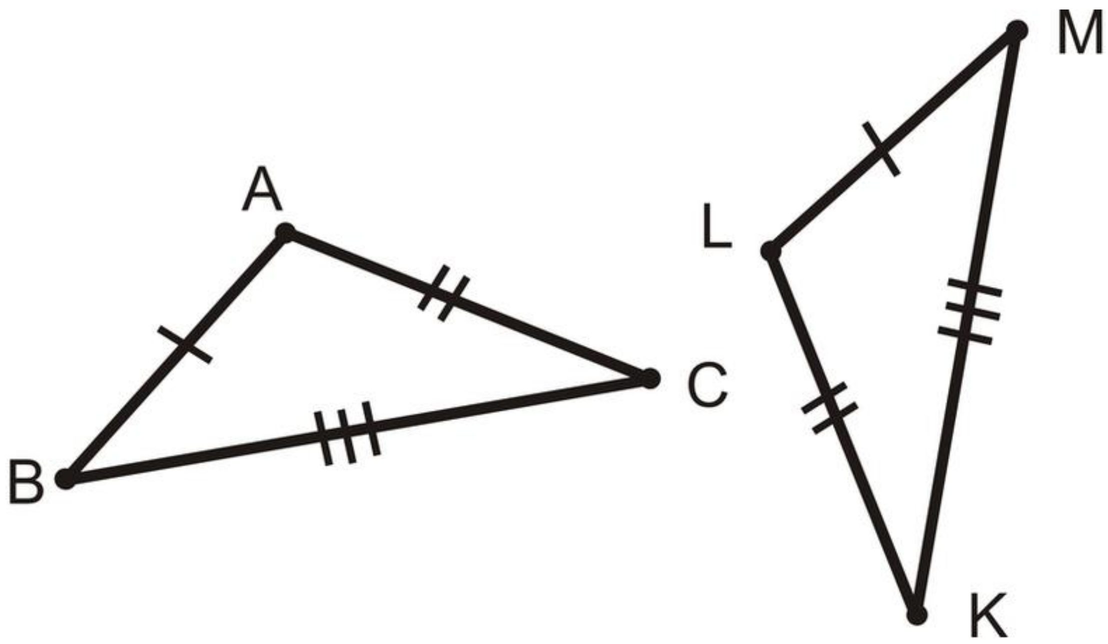
RHS (Right angle-Hypotenuse-Side)

Criteria for Congruency

SSS Criteria for Congruency

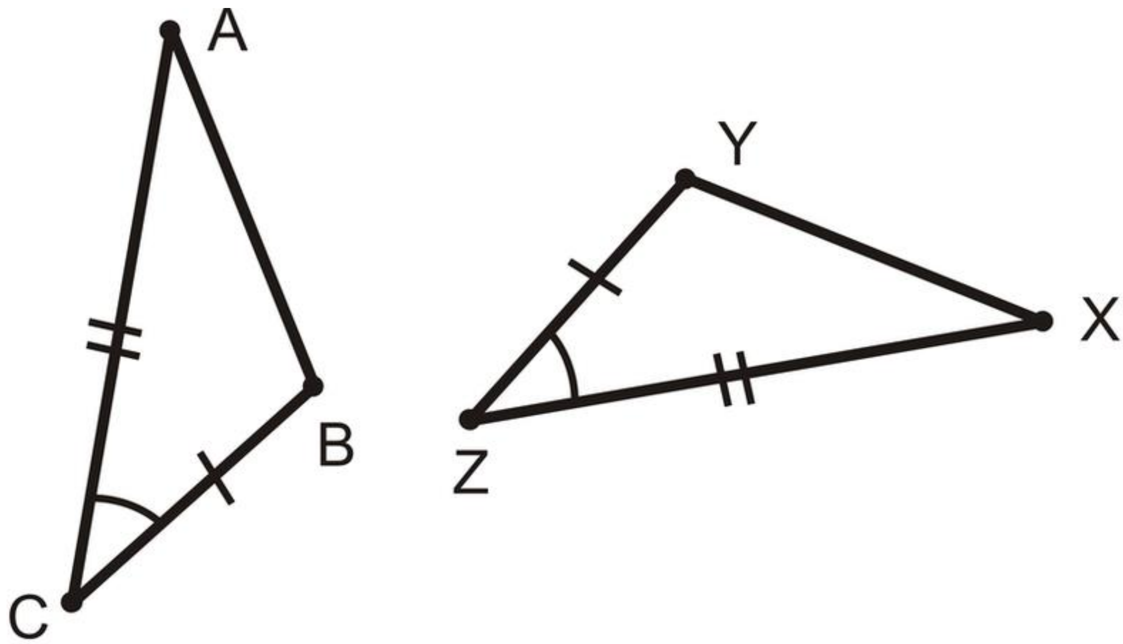
If under a given correspondence, the three sides of one triangle are equal to the three corresponding sides of another triangle, then the triangles are congruent.

Theorem: In two triangles, if the three sides of one triangle are equal to the corresponding three sides (SSS) of the other triangle, then the two triangles are congruent.



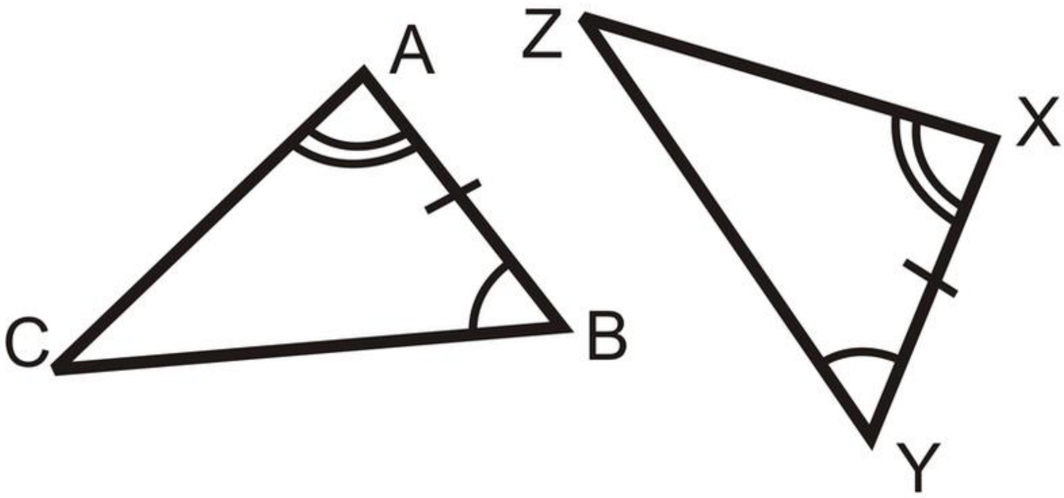
SAS Criteria for Congruency

If under a correspondence, two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent.



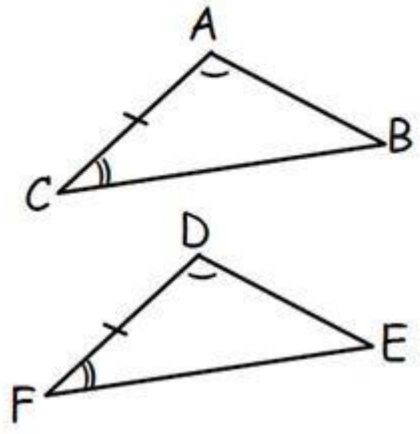
ASA Criteria for Congruency

If under a correspondence, two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.



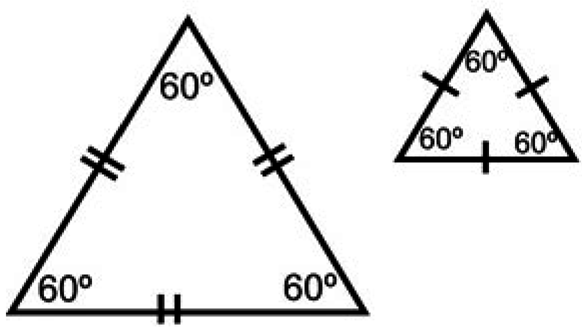
AAS Criteria for Congruency

AAS Rule: Triangles are congruent if two pairs of corresponding angles and a pair of opposite sides are equal in both triangles.

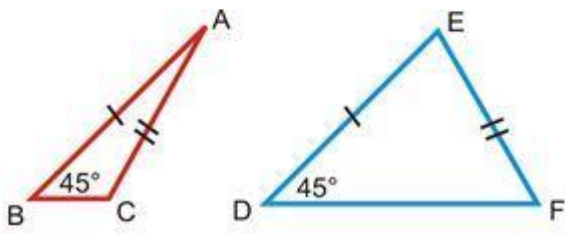


Why SSA and AAA congruency rules are not valid?

Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be an enlarged copy of the other. Therefore AAA congruency is not valid.



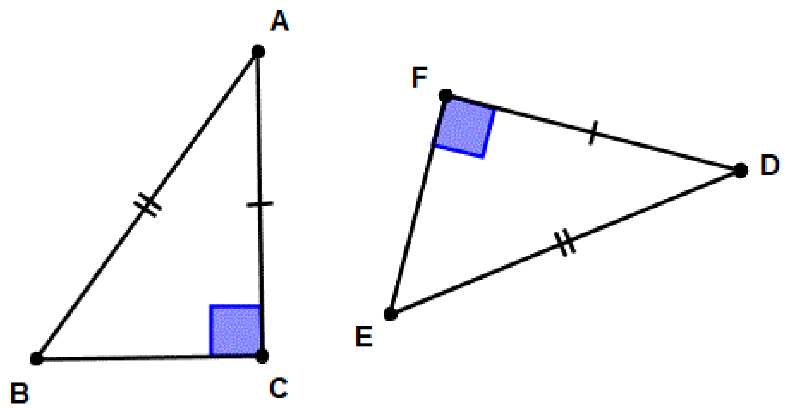
If two triangles have two congruent sides and a congruent non included angle, then triangles are not necessarily congruent. Therefore, SSA congruency is not valid.



RHS Criteria for Congruency

If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

Theorem: In two right-angled triangles, if the length of the hypotenuse and one side of one triangle, is equal to the length of the hypotenuse and corresponding side of the other triangle, then the two triangles are congruent.



Criteria for Congruency

Criteria for Congruency of two triangles are:

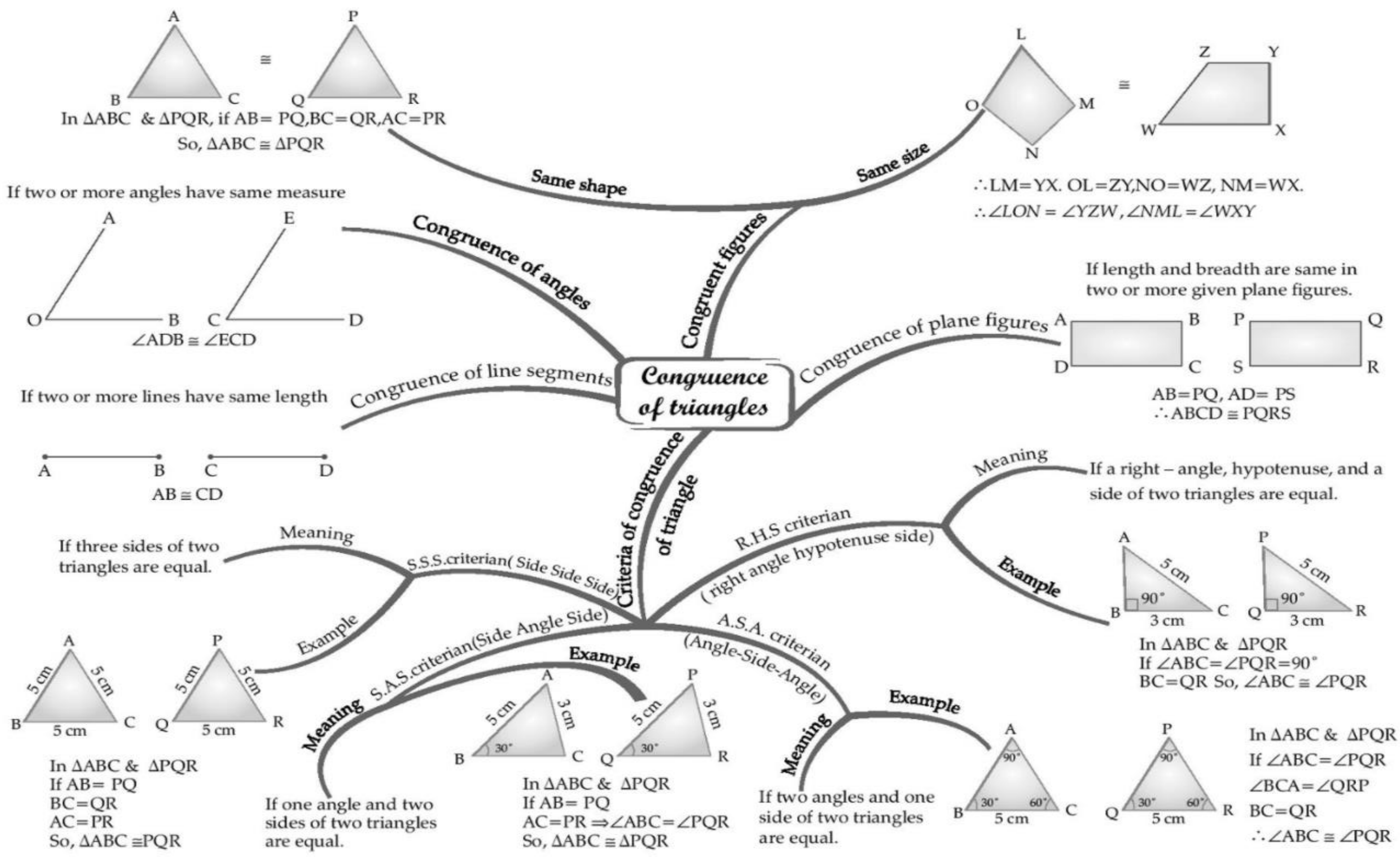
(i) SSS Rule

(ii) SAS Rule

(iii) ASA Rule

(iv) RHS Rule

CHAPTER - 7 CONGRUENCE OF TRIANGLES



Important Questions

Multiple Choice Questions :

Question 1. An angle is of 50° then its congruent angle is of:

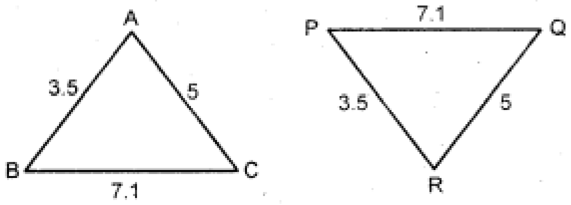
(a) 40°

(b) 60°

(c) 50°

(d) None of these

Question 2. Given two triangles are congruent then we can write :



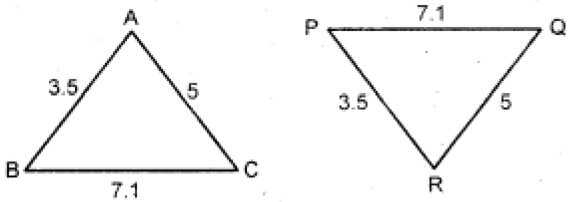
(a) ΔABC ≡ ΔPQR

(b) ΔABC ≡ ΔRPQ

(c) ΔABC ≡ ΔQRP

(d) none of these

Question 3. In the given figure, lengths of the sides of the triangles are given. Which pair of triangle are congruent ?



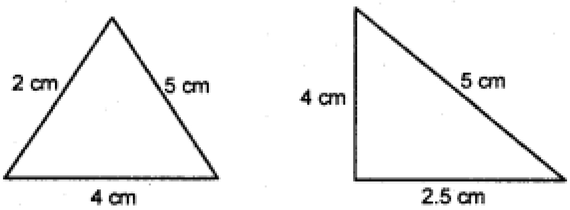
(a) ΔABC ≡ ΔPQR

(b) ΔBCA ≡ ΔPQR

(c) ΔABC ≡ ΔQRP

(d) none of these

Question 4. Are the following triangles congruent ?

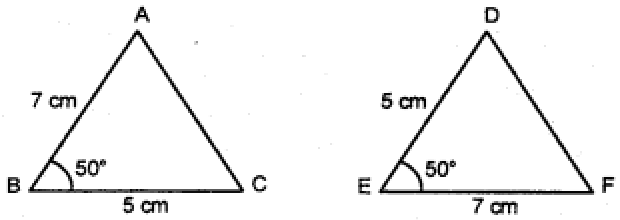


(a) yes

(b) no

(c) none of these

Question 5. Are the following triangles congruent ?

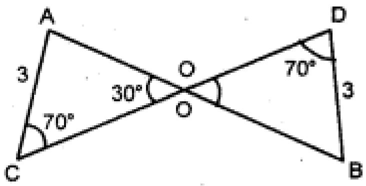


(a) yes

(b) no

(c) none of these

Question 6. In the given figure, say congruency of two triangles.



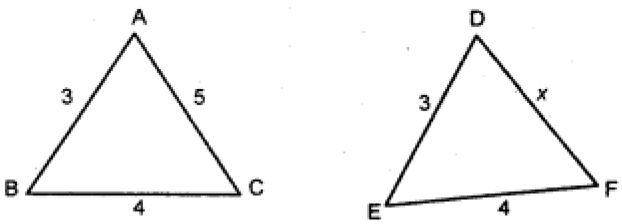
(a) ΔAOC ∪ ΔBOD

(b) ΔAOC ≠ ΔBOD

(c) ΔAOC ∪ ΔOBD

(d) none of these

Question 7. Given triangles are congruent, then what is the measurement of x ?



(a) 3

(b) 4

(c) 5

(d) none of these

Question 8. Given below are measurements of some parts of two triangles. Write the result in symbolic form.

In ΔABC, ∠B = 90°, AC 8 cm AB = 3 cm and

ΔPQR ,∠P = 90°, PR = 3 cm QR = 8 cm

(a) ΔABC ≡ ΔRPQ

(b) ΔABC ≡ ΔPQR

(c) ΔABC ≡ ΔRPQ

(d) none of these

Question 9. Given below are measurements of some parts of two triangles. Write the result in symbolic form if they are congruent.

In ΔABC,

∠A = 90°, AC = 5 cm, BC = 9 cm

In APQR,

∠P = 90°, PR = 3 cm QR = 8 cm

(a) are congruent

(b) are not congruent

Question 10. ΔABC and ΔPQR are congruent under the correspondence: ABC ↔ RPQ, then the part of ΔABC that correspond to PQ is

(a) AC

(b) AB

(c) BC

(d) None of These

Question 11. ΔABC is right triangle in which ∠A = 90° and AB = AC. The values of ∠B and ∠C will be

(a) ∠B = ∠C = 30°

(b) ∠B = ∠C = 50°

(c) ∠B = ∠C = 45°

(d) ∠B = ∠C = 60°

Question 12. Two students drew a line segment each. What is the condition for them to be congruent?

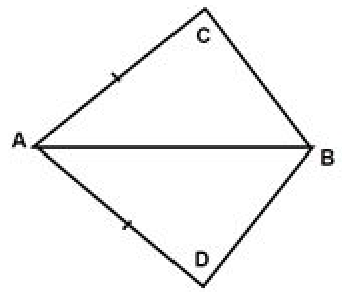
(a) They should be drawn with a scale.

(b) They should be drawn on the same sheet of paper.

(c) They should have different lengths.

(d) They should have the same length.

Question 13. In the quadrilateral ABCD, AC = AD and AB bisect ∠A and ΔABC ≅ ΔABD. The relation between BC and BD is



(a) BC < BD

(b) BC > BD

(c) BC = BD

(d) None of these

Question 14. A triangle in which all three sides are of equal lengths is called __________.

(a) Isosceles

(b) Equilateral

(c) Scalene

(d) None of these

Question 15. In ΔABC and ΔPQR, AB = 4 cm, BC = 5 cm, AC = 6 cm and PQ = 4 cm. QR = 5 cm. PR = 6 cm. then which of the following is true?

(a) ΔABC ≅ ΔQRP

(b) ΔABC ≅ ΔPQR

(c) ΔABC ≅ ΔRQP

(d) None of these

Very Short Questions :

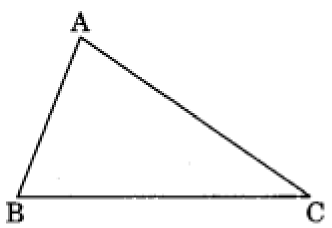
1. In the given figure, name

   (a) the side opposite to vertex A

   (b) the vertex opposite A to side AB

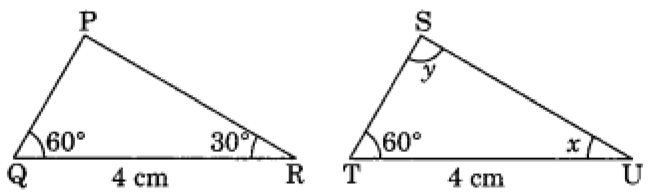
   (c) the angle opposite to side AC

   (d) the angle made by the sides CB and CA

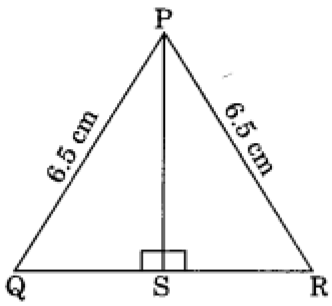


2. Examine whether the given triangles are congruent or not.

3. In the given congruent triangles under ASA, find the value of x and y, ΔPQR = ΔSTU.

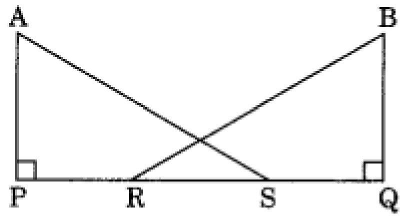


4. In the following figure, show that $\Delta PSQ = \Delta PSR$.



5. Can two equilateral triangles always be congruent? Give reasons.

6. In the given figure, AP = BQ, PR = QS. Show that $\Delta APS = \Delta BQR$



7. Without drawing the figures of the triangles, write all six pairs of equal measures in each of the following pairs of congruent triangles.

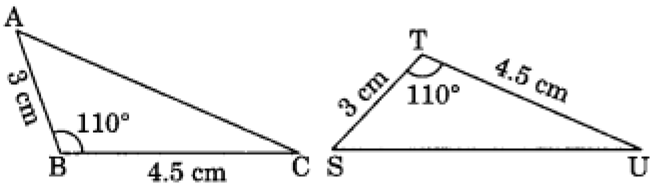
(i) ΔABC = ADEF

(ii) $\Delta XYZ = \Delta MLN$

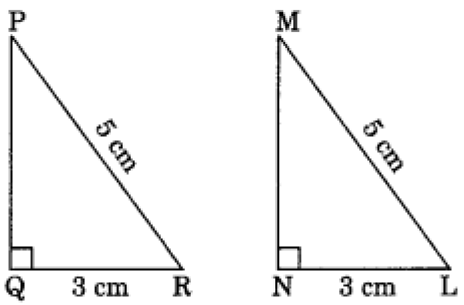
8. Lengths of two sides of an isosceles triangle are 5 cm and 8 cm, find the perimeter of the triangle.

Short Questions :

1. Write the rule of congruence in the following pairs of congruent triangles.

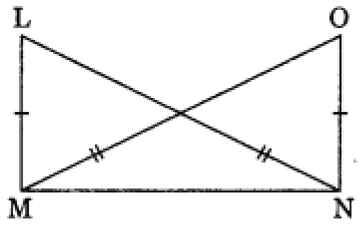


(i)



(ii)

2. In the given figure, state the rule of congruence followed by congruent triangles LMN and ONM.

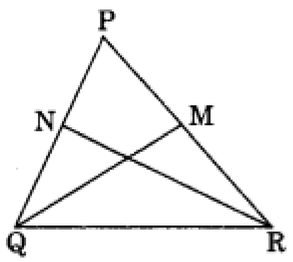


3. In the given figure, PQR is a triangle in which PQ = PR. QM and RN are the medians of the triangle. Prove that

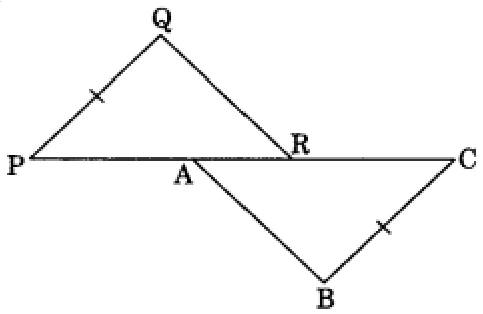
(i) ΔNQR = ΔMRQ

(ii) QM = RN

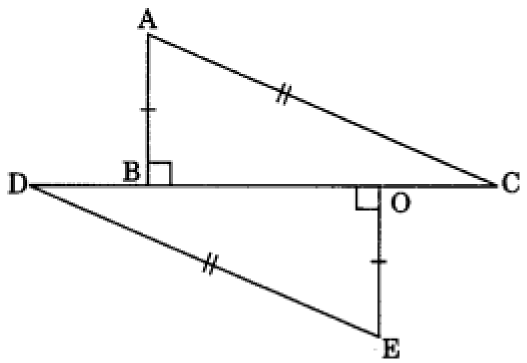
(iii) ΔPMQ = ΔPNR



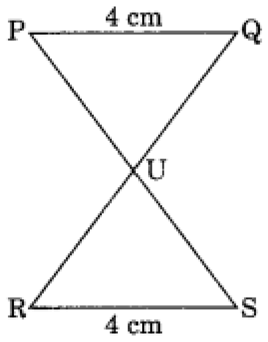
4. In the given figure, PQ = CB, PA = CR, ∠P = ∠C. Is ΔQPR = ΔBCA? If yes, state the criterion of congruence.



5. In the given figure, state whether ΔABC = ΔEOD or not. If yes, state the criterion of congruence.

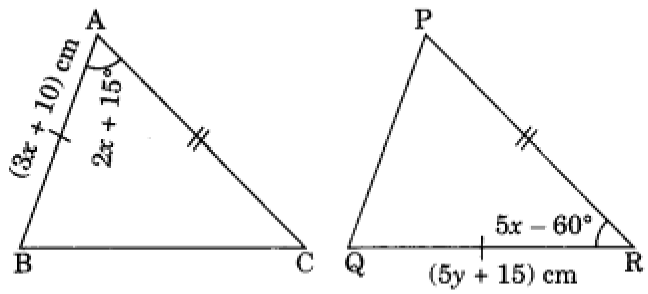


6. In the given figure, PQ || RS and PQ = RS. Prove that ΔPUQ = ΔSUR.



Long Questions :

1. In the given figure ΔBAC = ΔQRP by SAS criterion of congruence. Find the value of x and y.

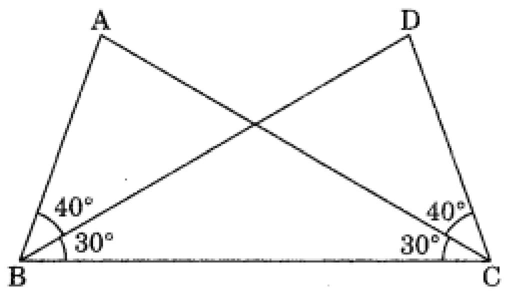


2. Observe the figure and state the three pairs of equal parts in triangles ABC and DCB.

(i) Is ΔABC = ΔDCB? Why?

(ii) Is AB = DC? Why?

(iii) Is AC = DB? Why?

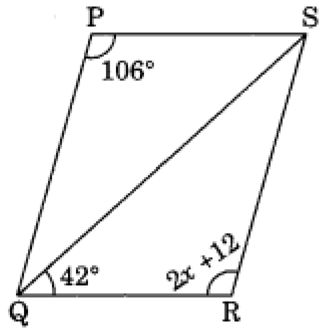


3. In the given figure, ΔQPS = ΔSRQ. Find each value.

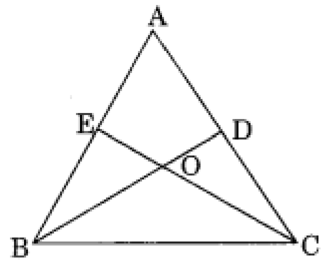
(a) x

(b) ∠PQS

(c) ∠PSR



4. In ΔABC, medians BD and CE are equal and intersect each other at O. Prove that ΔABC is an isosceles triangle.

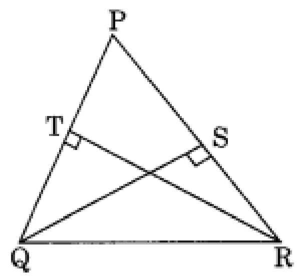


5. Prove that the lengths of altitudes drawn to equal sides of an isosceles triangle are also equal.

(i) ∠TRQ = ∠SQR?

(ii) If ∠TRQ = 30°, find the base angles of the ΔPQR.

(iii) Is ΔPQR an equilateral triangle?



ANSWER KEY -

Multiple Choice questions :

1. (c) 50°
2. (b) ΔABC ≡ ΔRPQ
3. (a) ΔABC ≡ ΔPQR
4. (b) no
5. (b) no
6. (a) ΔAOC ∪ ΔBOD
7. (c) 5
8. (a) ΔABC ≡ ΔRPQ

9. (b) are not congruent

10. (c) BC

11. (c) ∠B = ∠C = 45°

12. (d) They should have the same length.

13. (c) BC = BD

14. (b) Equilateral

15. (b) ΔABC ≅ ΔPQR

Very Short Answer :

1. (a) The side opposite to vertex A is BC.

(b) The vertex opposite to side AB is C.

(c) The angle opposite to side AB is ∠ACB.

(d) The angle made by the sides CB and CA is ∠ACB.

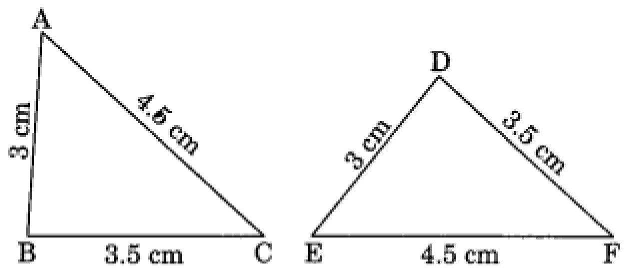
2. Here,

AB = DE = 3 cm

BC = DF = 3.5 cm

AC = EF = 4.5 cm

ΔABC = ΔEDF (By SSS rule)



So, ΔABC and ΔEDF are congruent.

3. Given: ΔPQR = ΔSTU (By ASA rule)

∠Q = ∠T = 60° (given)

$\overline{QR} = \overline{TU} = 4$ cm (given)

∠x = 30° (for ASA rule)

Now in ΔSTU,

∠S + ∠T + ∠U = 180° (Angle sum property)

∠y + 60° + ∠x = 180°

∠y + 60° + 30° = 180°

∠y + 90° = 180°

∠y = 180° – 90° = 90°

Hence, x = 30° and y = 90°.

4. In ΔPSQ and ΔPSR

$\overline{PQ} = \overline{PR} = 6.5$ cm (Given)

$\overline{PS} = \overline{PS}$ (Common)

∠PSQ = ∠PSR = 90° (Given)

ΔPSQ = ΔPSR (By RHS rule)

5. No, any two equilateral triangles are not always congruent.

Reason: Each angle of an equilateral triangle is 60° but their corresponding sides cannot always be the same.

6. In ΔAPS and ΔBQR

AP = BQ (Given)

PR = QS (Given)

PR + RS = QS + RS (Adding RS to both sides)

PS = QR

∠APS = ∠BQR = 90° (Given)

ΔAPS = ΔBQR (by SAS rule)

7. (i) Given: ΔABC = ΔDEF

Here AB = DE

BC = EF

AC = DF

∠A = ∠D, ∠B = ∠E and ∠C = ∠F

(ii) Given ΔXYZ = ΔMLN

Here XY = ML

YZ = LN

XZ = MN

∠X = ∠M, ∠Y = ∠L and ∠Z = ∠N

8. Since the lengths of any two sides of an isosceles triangle are equal, then

Case I: The three sides of the triangle are 5 cm, 5 cm and 8 cm.

Perimeter of the triangle = 5 cm + 5 cm + 8 cm = 18 cm

Case II: The three sides of the triangle are 5 cm, 8 cm and 8 cm.

Perimeter of the triangle = 5 cm + 8 cm + 8 cm = 21 cm

Hence, the required perimeter is 18 cm or 21 cm.

Short Answer :

1. (i) Here, AB = ST = 3 cm

BC = TU = 4.5 cm

$\angle ABC = \angle STU = 110°$

$\Delta ABC = \Delta STU$ (By SAS rule)

(ii) Here $\angle PQR = \angle MNL = 90°$

hypt. PR = hypt. ML

QR = NL = 3 cm

$\Delta PQR = \Delta MNL$ (By RHS rule)

2. In ΔLMN and ΔONM

LM = ON

LN = OM

MN = NM

$\Delta LMN = \Delta ONM$

3. ΔPQR is an isosceles triangle. [$\because$ PQ = PR]

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}PR$

$\Rightarrow$ NQ = MR and PN = PM

(i) In ΔNQR and ΔMRQ

NQ = MR (Half of equal sides)

$\angle NQR = \angle MRQ$ (Angles opposite to equal sides)

QR = RQ (Common)

$\Delta NQR = \Delta MRQ$ (By SAS rule)

(ii) QM = RN (Congruent parts of congruent triangles)

(iii) In ΔPMQ and ΔPNR

PN = PM (Half of equal sides)

PR = PQ (Given)

$\angle P = \angle P$ (Common)

$\Delta PMQ = \Delta PNR$ (By SAS rule)

4. PQ = CB, PA = CR

and $\angle P = \angle C$

In ΔQPR and ΔBCA,

PQ = CB (Given)

$\angle QPR = \angle BCA$ (Given)

PA = CR (Given)

PA + AR = CR + AR (Adding AR to both sides)

or PR = CA

ΔQPR = ΔBCA (By SAS rule)

5. In ΔABC and ΔEOD

AB = OE

∠ABC = ∠EOD = 90°

AC = ED

ΔABC = ΔEOD

Hence, ΔABC = ΔEOD

RHS is the criterion of congruence.

6. In ΔPUQ and ΔSUR

PQ = SR = 4 cm

∠UPQ = ∠USR (Alternate interior angles)

∠PQU = ∠SRU (Alternate interior angles)

ΔPUQ = ΔSUR (By ASA rule)

Long Answer :

1. Given: ΔBAC = ΔQRP (By SAS rule)

So, BA = QR

⇒ 3x + 10 = 5y + 15 ......(i)

∠BAC = ∠QRP

⇒ 2x + 15° = 5x – 60° ......(ii)

From eq. (ii), we have

2x + 15 = 5x – 60

⇒ 2x – 5x = -15 – 60

⇒ -3x = -7 5

⇒ x = 25

From eq. (i), we have

3x + 10 = 5y + 15

⇒ 3 × 25 + 10 = 5y + 15

⇒ 75 + 10 = 5y + 15

⇒ 85 = 5y + 15

$\Rightarrow 85 - 15 = 5y$

$\Rightarrow 70 = 5y$

$\Rightarrow y = 14$

Hence, the required values of x and y are 25 and 14 respectively.

2. (i) In ΔABC and ΔDCB

$\angle ABC = \angle DCB = 70° \ (40° + 30° = 70°)$ (Given)

$\angle ACB = \angle DCB = 30°$ (Given)

BC = CB (Common)

ΔABC = ΔDCB (By ASA rule)

(ii) Yes,

AB = DC (Congruent parts of congruent triangles)

(iii) Yes,

AC = DB (Congruent parts of congruent triangles)

3. (a) ΔQPS = ΔSRQ

$\angle QPS = \angle SRQ$ (Congruent part of congruent triangles)

$106 = 2x + 12$

$\Rightarrow 106 - 12 = 2x$

$\Rightarrow 94 = 2x$

$\Rightarrow x = 47$

$\angle QRS = 2 \times 47 + 12 = 94 + 12 = 106°$

So, PQRS is a parallelogram.

$\angle QSR = 180° - (42° + 106°) = 180° - 148° = 32°$

(b) $\angle PQS = 32°$ (alternate interior angles)

(c) $\angle PSQ = 180° - (\angle QPS + \angle PQS) = 180° - (106° + 32°) = 180° - 138° = 42°$

$\angle PSR = 32° + 42° = 74°$

4. We know that the medians of a triangle intersect each other in the ratio 2 : 1.

BD = CE (Given)

$$\frac{2}{3}\text{BD} = \frac{2}{3}\text{CE}$$

$\Rightarrow$ OB = OC

$$\frac{1}{3}\text{BD} = \frac{1}{3}\text{CE}$$

$\Rightarrow$ OE = OD

In ΔBOE and ΔCOD,

OB = OC

OE = OD

∠BOE = ∠COD (Vertically opposite angles)

ΔBOE = ΔCOD (By SAS rule)

BE = CD (Congruent parts of congruent triangles)

2BE = 2CD

⇒ AB = AC

Hence ΔABC is an isosceles triangle.

5. In ΔQTR and ΔRSQ

∠QTR = ∠RSQ = 90° (Given)

∠TQR = ∠SRQ (Base angle of an isosceles triangle)

∠QRT = ∠RQS (Remaining third angles)

QR = QR (Common)

ΔQTR = ΔRSQ (By ASA rule)

QS = RT (Congruent parts of congruent triangles)

Hence proved.

(i) ∠TRQ = ∠SQR (Congruent parts of congruent triangles)

(ii) In ΔQTR,

∠TRQ = 30° (Given)

∠QTR + ∠TQR + ∠QRT = 180° (Angle sum property)

⇒ 90° + ∠TQR + 30° = 180°

⇒ 120° + ∠TQR = 180°

⇒ ∠TQR = 180° – 120° = 60°

⇒ ∠TQR = ∠SRQ = 60°

Each base angle = 60°

(iii) In ΔPQR,

∠P + ∠Q + ∠R = 180° (Angle sum property)

⇒ ∠P + 60° + 60° = 180° (From ii)

⇒ ∠P + 120° = 180°

⇒ ∠P = 180° – 120° = 60°

Hence, ΔPQR is an equilateral triangle.