MATHEMATIC

Chapter 8: Comparing Quantities



Comparing Quantities

Fractions and Ratios

A fraction represents a part of a whole which consists of numerators and denominators and it is the division of two same quantities.

Eg:
$$\frac{3}{5}$$

Ratio is the comparison of one value to the other or the comparison of two different quantities.

Eg: 3:5

Finding the Increase or Decrease in Percent

Finding Increase or Decrease Percentage in Situations

Finding new number, when there is increase in percentage.

New number = original number + (increase in percentage × number)

Ex: The Cost of a mobile phone is Rs 15,000. Find the new price if there is a increase of 5%

New price = original price + 5% of original price

New price =
$$15,000 + \frac{5}{100} \times 15,000$$

New price =
$$15,000 + 750 = 15,750$$

Here Rs 750 is increase in the price.

The new number can be found out using,

New number = original number × percentage increase

E.g.: New price =
$$15,000 \times 105 \div 100 = 15,000 \times 1.05 = 15,750$$

Finding new number, when there is decrease in percentage.

New number = original number – (decrease in percentage × number)

Also, New number = original number × percentage decrease

E.g.: The Cost of a mobile phone is Rs 15,000. Find the new price if there is a decrease of 5%

New price =
$$15,000 \times 95 \div 100 = 15,000 \times 0.95 = 14,250$$

Finding SP without Finding Discount Percentage

A reduction (decrease) on the marked price is known as discount.

If the discount is given in numbers, then it is calculated by

Discount = Marked price - Sale price

If the discount is given in percentage, then it is calculated by

Discount = Discount % of Marked price

Finding Discounts

If the discount is given in numbers.

Example: Marked price of a shirt is Rs 535. Its selling price is Rs 495. Find the discount.

Solution: Discount = Marked price - Sale price

Discount = Rs 535 - Rs 495 = Rs 40

If the discount is given in percentage.

Example: A toy priced Rs 500 is available at a discount of 5%. Find the discount.

Solution: Discount = Discount % of Marked price

Discount = 5% of 500 = $\frac{5}{100} \times 500$

Discount = Rs 25

Estimation of Amounts (In Percentages)

Estimating amounts when there is a discount or hike on the marked price.

Example: Anil bought a pair of shoes priced Rs 650, at a discount of 10%. Find the billing amount.

Solution: Billing amount = Marked price – discount

Billing amount = Rs 650 - $\frac{10}{100}$ × 650

Billing amount = Rs 650 - Rs 65 = Rs 585

Example: Shilpa bought a new mobile for Rs 15,000. She has to pay 2% as delivery charges.

Find the billing amount.

Solution: Billing amount = Marked price + Hike

Billing amount = Rs 15,000 +
$$\frac{2}{100}$$
 × 15000

Billing amount = Rs 15,000 + Rs 300 = Rs 15,300

Profit and Loss

Let us learn profit and loss concepts in maths. It is well explained in terms of cost price and selling price.

Profit(P): The amount gained by selling a product with more than its cost price.

Loss(L): The amount the seller incurs after selling the product less than its cost price, is mentioned as a loss.

Cost Price (CP): The amount paid for a product or commodity to purchase it is called a cost price. Also, denoted as CP. This cost price is further classified into two different categories:

Fixed Cost: The fixed cost is constant, it doesn't vary under any circumstances

Variable Cost: It could vary depending as per the number of units

Selling Price (SP) The amount for which the product is sold is called Selling Price. It is usually denoted as SP. Also, sometimes called a sale price.

Marked Price Formula (MP) This is basically labelled by shopkeepers to offer a discount to the customers in such a way that,

Discount = Marked Price - Selling Price

And Discount Percentage = (Discount/Marked price) x 100

Profit and Loss Formulas

Now let us find profit formula and loss formula.

The profit or gain is equal to the selling price minus cost price.

Loss is equal to cost price minus selling price.

Profit or Gain = Selling price – Cost Price

Loss = Cost Price – Selling Price

The formula for the profit and loss percentage is:

Profit percentage = (Profit /Cost Price) x 100

Loss percentage = (Loss / Cost price) x 100

Prices / Charges Related to Buying and Selling

Profit = Selling price - Cost price

Profit % =
$$\frac{Profit}{Cost\ price} \times 100$$

Loss = Cost price - Selling price

Loss % =
$$\frac{Loss}{Cost\ price}$$
 ×100

Finding Prices / Charges Related to Buying and Selling

Example: A shopkeeper sold a T.V priced Rs 12,000 at Rs 13,500. Find his profit percentage.

Profit = Selling price – Cost price

Profit = Rs 13,500 - Rs 12,000 = Rs 1,500

Profit
$$\% = \frac{Profit}{Costprice} \times 100$$

Profit
$$\% = \frac{1500}{12000} \times 100 = 12.5\%$$

Example: Amit sold his laptop, priced Rs 20,000 at Rs 18,000. Find his loss percentage.

Loss = Cost price – Selling price

Loss = Rs 20,000 - Rs 18,000 = Rs 2000

$$Loss \% = \frac{Loss}{Costprice} \times 100$$

Loss% =
$$\frac{2000}{20,000} \times 100 = 10\%$$

Sales Tax / VAT

Sales tax or value added tax(VAT) is the tax that should be paid to the government on sale of an item

and it is added to the bill amount.

Normally, VAT is included in the price of items like groceries.

Finding Sales Tax / VAT

Sales tax or VAT = Tax % of Selling price

Billing Amount = Selling price + VAT

Example: Megha bought a wrist watch for Rs 1,200 and VAT is charged at 8%. Calculate the VAT and billing amount.

Solution: VAT = Tax % of selling price

VAT = 8% of 1,200 =
$$\frac{8}{100}$$
 × 1200 = Rs 96

Billing amount = S.P + VAT = Rs 1,200 + Rs 96 = Rs 1296.

Simple and Compound Interest

SI

Simple interest is the extra money charged on a loan where the principal amount will be fixed for a particular time period.

Interest is the extra money that a bank gives for saving or depositing money with them.

Similarly, when anybody borrow money, they pay interest.

Simple interest =
$$\frac{P.T.R}{100}$$
, where

P is the principal amount

T is the number of years.

R is the interest rate

Calculating CI

Compound interest is the interest, calculated on the principal and the interest for the previous period.

The principal amount increases with every time period, as the interest payable is added to the principal.

Eg: Find CI on Rs 10,000 for 2 years at an interest rate of 5%.

Ans: Interest for the 1st year

For 1st year,
$$P = 10,000$$
, $T = 1$ year, $R = 5\%$

$$I_1 = \frac{P.T.R}{100} = \frac{10000.1.5}{100} = \text{Rs } 500$$

$$A = P + I_1 = 10,000 + 500 = 10,500$$

Interest for the 2nd year

For 2nd year,
$$P = 10,500$$
, $T = 1$ year, $R = 5\%$

$$I_2 = \frac{P.T.R}{100} = \frac{10500.1.5}{100} = \text{Rs } 525$$

$$C.I = I_1 + I_2 = Rs 500 + Rs 525 = Rs 1025$$

Deducing a Formula for Compound Interest

Formula for CI

Calculation of compound interest can be generalized.

let P1 be the sum on which the interest is compounded annually at the rate of R

Then the interest for the 1st year,

$$|_1 = \frac{P_1.1.R}{100}$$

$$=\frac{P_1.R}{100}$$

$$A_1 = P_1 + I_1 = P_1 + \frac{P_1 \cdot R}{100}$$

$$A_1 = P_1 \left(1 + \frac{R}{100} \right) = P_2$$

For 2nd year,

$$\begin{aligned} &\mathsf{P}_2 = \mathsf{P}_1 \big(1 + \frac{R}{100} \big), \mathsf{T} = \mathsf{1} \text{ year and R=R}\% \\ &\mathsf{I}_2 = \frac{P_2.1.R}{100} = \frac{P_2.R}{100} \\ &\mathsf{I}_2 = \mathsf{P}_1 \big(1 + \frac{R}{100} \big) \times \frac{R}{100} \\ &\mathsf{I}_2 = \frac{P_1R}{100} \big(1 + \frac{R}{100} \big) \end{aligned}$$

$$|_2 = \frac{P_2.1.R}{100} = \frac{P_2.R}{100}$$

$$I_2 = P_1 \left(1 + \frac{R}{100}\right) \times \frac{R}{100}$$

$$I_2 = \frac{P_1 R}{100} (1 + \frac{R}{100})$$

$$A_2 = P_2 + I_2$$

$$A_2 = P_1 \left(1 + \frac{R}{100}\right) + \frac{P_1 R}{100} \left(1 + \frac{R}{100}\right)$$

$$\begin{array}{l} \mathsf{A_2} \! = \! \mathsf{P_1} (1 \! + \! \frac{R}{100}) \! + \! \frac{P_1 R}{100} (1 \! + \! \frac{R}{100}) \\ \mathsf{A_2} \! = \! \mathsf{P_1} (1 \! + \! \frac{R}{100}) (1 \! + \! \frac{R}{100}) \; [\text{taking P}_1 (1 \! + \! \frac{R}{100}) \; \text{as common}] \; \mathsf{A_2} \! = \! \mathsf{P_1} (1 \! + \! \frac{R}{100})^2 \end{array}$$

Where, P is the principal amount, R is the rate of interest and n is the number of years.

We get the formula for the amount to be paid at the end of n years.

Compound Interest can be calculated using the formula,

$$CI = A - P$$

Rate Compounded Annually or Half-Yearly

If interest is compounded annually,

time span, n = 1 year, here the principal amount varies yearly.

Principal amount (A = P + I_1) for first year will serve as the principal for the second year.

If interest is compounded half – yearly,

time span, n = 6 months, here the principal amount varies half – yearly.

Principal amount (A = P + I_1) for first 6 months will be the principal for the next 6 months.

Finding CI When Rate Compounded Annually or Semi – Annually

When compound interest is compounded annually,

$$A=P(1+\frac{R}{100})^n$$

Where, P is the principal amount, R is the rate of interest and n is the number of years.

When compound interest is compounded half yearly,

the interest rate will be half of the annual interest rate and the time period will be doubled.

$$A=P(1+\frac{R}{200})^{2n}$$

Where, P is the principal amount, R is the rate of interest and n is the number of years.

Application of Compound Interest

Application of compound interest are:

To calculate the growth rate of population (increase or decrease).

To calculate change in the price of an item (increase or decrease).

Example: If the population of a town increases 2% annually and the present population is 3,26,40,000, find its population after 2 years.

Solution.
$$P = 3,26,40,000$$
 $n = 2$ years, $R = 2\%$

Therefore, Population after 2 years

$$A=P(1+\frac{R}{100})^n$$

$$A=32640000(1+\frac{2}{100})^2$$

A=32640000×
$$(\frac{51}{50})^2$$

A=32640000
$$\times \frac{51}{50} \times \frac{51}{50}$$

∴ The population after 2 years is 3,39,58,656

Example: A motorcycle is bought at Rs 1,60,000. Its value depreciates at the rate of 10% per annum. Find its value after 2 years.

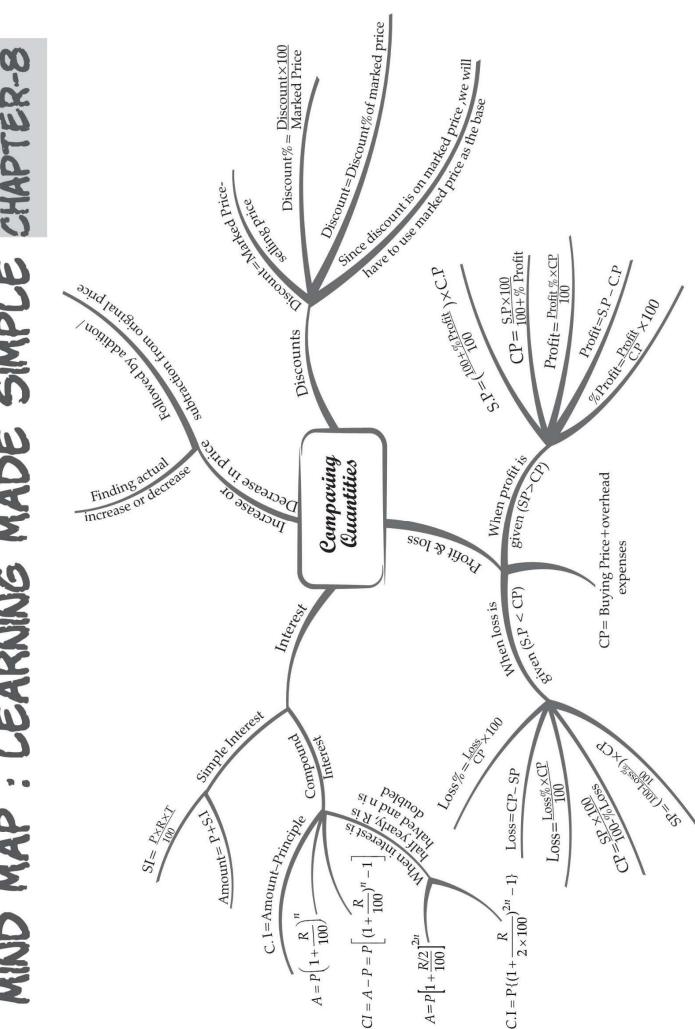
Solution. P = 1,60,000 n = 2 years, R = 10%

$$A=P(1-\frac{R}{100})^n$$

$$A=160000\times(1-\frac{10}{100})^2$$

$$A=160000 \times \frac{9}{10} \times \frac{9}{10}$$

MIND MAP: LEARNING MADE SIMPLE CHAPTER-8



Important Questions

Multiple Choice Questions-

Question 1. A shopkeeper purchased 300 bulbs for Rs 10 each. However 10 bulbs were fused and had to be thrown away. The remaining were sold at Rs 12 each. Find the gain or loss %.

- (a) 15%
- (b) 13%
- (c) 16%
- (d) none of these

Question 2. Find the ratio of 5 m to 10 km.

- (a) It is 1:3
- (b) It is 3000:1
- (c) It is 2000:1
- (d) It is 1:2000

Question 3. Rohan bought a second hand refrigerator for Rs 2,500, then spent Rs 500 on its repairs and sold it for Rs 3,300. Find his loss or gain per cent.

- (a) Loss 15% 2a
- (b) Loss 10%
- (c) Profit 10%
- (d) None of these

Question 4. The sale price of a shirt is Rs.176. If a discount of 20% is allowed on its marked price, what is the marked price of the shirt?

- (a) Rs.160
- (b) Rs.180
- (c) Rs. 200
- (d) Rs. 220

Question 5. Find selling price (SP) if a profit of 5% is made on a cycle of Rs 700 with Rs 50 as overhead charges.

- (a) Rs 600
- (b) Rs 787.50
- (c) Rs 780
- (d) None of these

Question 6. Sohan bought a washing machine for Rs 40,000, then spent Rs 5,000 on its repairs and sold it for Rs 50,000. Find his loss or gain per cent.

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- (b) Loss 20%

(a) Loss 10%

- (c) Profit 11%
- (d) none of these

Question 7. Find the ratio of speed of a cycle 20 km per hour to the speed of scooter 30 km per hour.

- (a) It is 3:1
- (b) It is 2:1
- (c) It is 2:3
- (d) It is 1:3

Question 8. A shopkeeper purchased 500 pieces for Rs 20 each. However 50 pieces were spoiled in the way and had to be thrown away. The remaining were sold at Rs 25 each. Find the gain or loss %.

- (a) 18%
- (b) 15%
- (c) 12.5%
- (d) none of these

Question 9. A picnic is being planned in a school. Girls are 60% of the total number of students and are 300 in number. Find the ratio of the number of girls to the number of boys in the class.

- (a) It is 3:2
- (b) It is 3:1
- (c) It is 2:3
- (d) It is 2:1

Question 10. An item marked at Rs 840 is sold for Rs 714. What is the discount %?

- (a) 20%
- (b) 10%
- (c) 15%
- (d) none of these

Very Short Questions:

- **1.** Express the following in decimal form:
 - (a) 12%
 - (b) 25%

- 2. Evaluate the following:
 - (a) 20% of 400
 - (b) $12\frac{1}{2}\%$ of 625
- If 20% of x is 25, then find x. 3.
- Express the following as a fraction 4.
 - (a) 35%
 - (b) 64%
- 5. Express the following into per cent
 - (a) $1\frac{3}{5}$
 - (b) 2:5
- There are 24% of boys in a school. If the number of girls is 456, find the total number 6. of students in the school.
- The cost of 15 articles is equal to the selling price of 12 articles. Find the profit per **7.** cent.
- An article is marked at ₹ 940. If it is sold for ₹ 799, then find the discount per cent. 8.

Short Questions:

- A watch was bought for ₹ 2,700 including 8% VAT. Find its price before the VAT 1. was added.
- Find the amount if ₹ 2,000 is invested for 2 years at 4% p.a. compounded 2. annually.
- 3. A number is increased by 20% and then it is decreased by 20%. Find the net increase or decrease per cent.
- Two candidates Raman and Rajan contested an election. Raman gets 46% of the 4. valid votes and is defeated by 1600 votes. Find the total number of valid votes cast in the election.
- 5. A man whose income is ₹ 57,600 a year spends ₹ 43,200 a year. What percentage of his income does he save?
- A CD player was purchased for ₹ 3,200 and ₹ 560 were spent on its repairs. It 6. was then sold at a gain of $12\frac{1}{2}\%$ How much did the seller receive?
- A car is marked at ₹ 3,00,000. The dealer allows successive discounts of 6%, 4% **7.** and $2\frac{1}{2}$ % on it. What is the net selling price of it?
- Ramesh bought a shirt for ₹ 336, including 12% ST and a tie for ₹ 110 including 8. 10% ST. Find the list price (without sales tax) of the shirt and the tie together.

Long Questions:

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- 1. Find the amount of ₹ 6,250 at 8% pa compounded annually for 2 years. Also, find the compound interest.
- 2. Find the compound interest on ₹ 31,250 at 12% pa for $12\frac{1}{2}$ years.
- 3. Vishakha offers a discount of 20% on all the items at her shop and still makes a profit of 12%. What is the cost price of an article marked at ₹ 280?
- **4.** Find the compound interest on ₹ 48,000 for one year at 8% per annum when compounded half yearly.
- Sahana sells two watches for Rs. 1955 each gaining 15% on one and losing
 15% on the other. Find her gain or loss percent in the whole transaction.

Answer Key-

Multiple Choice questions-

1. (c) 16%

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- **2.** (d) It is 1:2000
- 3. (c) Profit 10%
- 4. (d) Rs. 220
- **5.** (b) Rs 787.50
- 6. (c) Profit 11%
- **7.** (c) It is 2:3
- **8.** (c) 12.5%
- **9.** (a) It is 3:2
- **10.** (c) 15%

Very Short Answer:

- 1. (a) $12\% = \frac{12}{100} = 0.12$
 - (b) $25\% = \frac{25}{100} = 0.25$
- 2.
- (a) 20% of 400 = $\frac{20}{100} \times 400 = 80$
- (b) $12\frac{1}{2}\%$ of $625 = \frac{25}{2}\%$ of 625= $\frac{25}{2 \times 100_4} \times 625 = \frac{625}{8} = 78\frac{1}{8}$

$$\Rightarrow \frac{20}{100} \times x = 25$$

$$\therefore x = 25 \times \frac{100}{20} = 125$$

Hence x = 125

4.

$$(a) 35\% = \frac{35}{100} = \frac{7}{20}$$

(b)
$$64\% = \frac{64}{100} = \frac{16}{25}$$

5.

(a)
$$1\frac{3}{5} = \frac{8}{5} = \frac{8}{5} \times 100 = 160\%$$

(b)
$$2:5=\frac{2}{5}=\frac{2}{5}\times 100=40\%$$

6. Let the total number of students be 100.

Number of boys = 24% of
$$100 = \frac{24}{100} \times 100 = 24$$

Number of girls = 100 - 24 = 76

 \Rightarrow If number of girls is 76, then total number of students = 100

 \Rightarrow If Number of girls is 1, then total number of students = 10076

If Number of girls is 456, then total number of students $=\frac{100\times45}{676}=600$

Hence, the total number of students in the school = 600

7.

∴ Profit on 1 article = ₹
$$\frac{100}{12}$$
 - ₹ $\frac{100}{15}$
= ₹ $\left(\frac{500 - 400}{60}\right)$
= ₹ $\frac{100}{60}$

$$\therefore Profit per cent = \frac{Profit}{CP} \times 100$$

$$= \frac{\frac{100}{60}}{\frac{100}{15}} \times 100$$

8. Hence, profit = 25%

Discount = MP – SP =
$$940 - 799 = ₹ 141$$

$$\therefore \text{ Discount per cent} = \frac{\text{Discount}}{\text{MP}} \times 100$$
$$= \frac{\cancel{141}^3}{\cancel{41}\cancel{94\cancel{9}}} \times \cancel{\cancel{100}} = 15\%$$

Hence, discount = 15%

Short Answer:

1. Cost of watch including VAT = ₹ 2,700

Let the initial cost of the watch be ₹ 100

Cost of watch including VAT = ₹ 100 + ₹8 = ₹ 108

If cost including VAT is ₹ 108, then its initial cost = ₹ 100

If cost including VAT is $\stackrel{?}{=}$ 1, then its initial cost = $\stackrel{?}{=}$ $\frac{100}{108}$

If cost including VAT is $\stackrel{?}{\underset{?}{?}}$ 2,700, then its initial cost $= \stackrel{?}{\underset{108}{?}} \times 2700 = \stackrel{?}{\underset{?}{?}} 2500$

Hence, the required cost = ₹ 2,500

2.

$$A = P \left(1 + \frac{R}{100} \right)^n$$

$$= 2000 \left(1 + \frac{4}{100} \right)^2$$

$$= 2000 \times \left(\frac{26}{25}\right)^2$$

$$= 2000 \times \frac{26 \times 26}{25 \times 25}$$

$$= \frac{16 \times 676}{5} = \text{ } \frac{10,816}{5}$$
$$= \text{ } \text{ } 2,163.20$$

Hence, the required amount = ₹ 2,163.20

3. Let the number be 100

20% increase =
$$\frac{20}{100} \times 100 = 20$$

Increased value = 100 + 20 = 120

Now it is decreased by 20%

Decreased value =
$$120 - \frac{120}{100} \times 20 = 120 - 24 = 96$$

Net decrease =
$$100 - 96 = 4$$

Decrease per cent =
$$\frac{4}{100} \times 100 = 4\%$$

Hence, the net decrease per cent = 4%

4. Let the total number of valid votes be 100

Number of votes got by Raman = 46% of
$$100 = \frac{46}{100} \times 100 = 46$$

Number of votes got by Rajan = 100 - 46 = 54

Difference between the votes = 54 - 46 = 8

8% of Valid votes = 1,600

$$\Rightarrow \frac{8}{100} \times \text{Valid votes} = 1,600$$

$$\Rightarrow$$
 Valid votes = $\frac{1600 \times 100}{8}$ = 20,000

Hence, the total number of valid votes = 20,000

5. Annual income of a man = ₹ 57,600

Amount spent by him in the year = ₹43,200

Net amount saved by him = ₹ 57,600 - ₹ 43,200 = ₹ 14,400

Percentage of his annual saving Saving = $\frac{Saving}{Income} \times 100$

$$= \frac{14400}{57600} \times 100$$
$$= 25\%$$

Hence, the saving percentage = 25%

Cost price of the CD player = ₹ 3,200 6.

Amount spent on its repairing = ₹ 560

Net cost price = ₹3,200 + ₹560 = ₹3,760

SP = CP
$$\left(1 + \frac{\text{gain}}{100}\right)$$

= 3,760 $\left(1 + \frac{25}{2 \times 100}\right)$
= 3,760 × $\frac{9}{8}$
= 470 × 9 = ₹ 4,230

Hence, the required amount = ₹4,230

7. Marked price of the car = ₹ 3,00,000
Net selling price after the successive discounts

= 3,00,000×
$$\left(\frac{100-6}{100}\right)$$

× $\left(\frac{100-4}{100}\right)$ × $\left(\frac{100-2.5}{100}\right)$
= 3,00,000× $\frac{94}{100}$ × $\frac{96}{100}$ × $\frac{97.5}{100}$
= $\frac{3\times94\times96\times97.5}{10}$ = ₹ 2,63,952

Hence, the net selling price = ₹ 2,63,952

8. List price of the shirt $=\frac{110}{112} \times 336 = ₹300$ List price of the tie $=\frac{100}{110} \times 110 = ₹100$ List price of both together = ₹300 + ₹100 = ₹400

Long Answer:

1.

A = P
$$\left(1 + \frac{R}{100}\right)^n$$

= 6,250 $\left(1 + \frac{8}{100}\right)^2$
= 6,250 $\left(\frac{27}{25}\right)^2$
= 6,250× $\frac{27}{25}$ × $\frac{27}{25}$ = ₹ 7,290
∴ CI = A - P
= ₹ 7290 - ₹6,250 = ₹ 1,040
Hence, amount = ₹ 7,240 and CI = ₹ 1,040

A = 31,250
$$\left(1 + \frac{12}{100}\right)^2 \left(1 + \frac{12 \times \frac{1}{2}}{100}\right)$$

= 31,250 × $\frac{28}{25}$ × $\frac{28}{25}$ × $\frac{53}{50}$
= 31,250 × $\frac{784}{625}$ × $\frac{53}{50}$ = ₹ 41,552
CI = A - P
=₹ 41,552 - ₹ 31,250
=₹ 10,302

Hence, compound interest = ₹ 10,302

$$=\frac{1}{2}$$
 × 280 = ₹ 56

Let the cost price be ₹ 100

If the selling price is ₹ 112, cost price = ₹ 100

If the selling price is ₹ 224, cost price = ₹
$$(\frac{100}{112} \times 224)$$
 = ₹ 200

Time
$$(n) = 1$$
 year

Interest is compounded half yearly

A = P
$$\left(1 + \frac{R}{200}\right)^{2n}$$

= 48,000 $\left(1 + \frac{8}{200}\right)^{2}$
= 48,000 × $\frac{26}{25}$ × $\frac{26}{25}$
= 76.8 × 26 × 26
= ₹ 51,916.80

Therefore Compound Interest = A - P = ₹ (519,16.80 - 48,000) = ₹ 3,916.80.

5. CP = Rs. 1955. Gain = 15%
SP =
$$1955 \times \frac{115}{100} = Rs. 2248.25$$

CP = Rs. 1955

SP =
$$1955 \times \frac{85}{100}$$
 = $Rs.1661.75$
CP = Rs. 1955 + Rs. 1955 = Rs. 3910
SP = Rs. 2248.25 + Rs. 1661.75 = 3910
CP > SP Loss of Rs(3990 - 3910) = Rs. 80 = 2%
Loss percent = 2% .